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**A Tale of Two Garnets: The Role of Solid Solution in  
the Development Towards a Modern Mineralogy**

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## ABSTRACT

This article reviews the development of mineralogy as a science by focusing largely on the common silicate garnets of general formula  $\{X_3\}[Y_2](Si_3)O_{12}$ . It tells of a number of important discoveries, analyses and proposals by various scientists relating to crystallography, crystal structures, isomorphism and solid solution starting in Europe in the late 1700s. The critical recognition of the importance of ionic size in determining crystal-chemical properties and solid-solution behavior is emphasized. The two garnet species “pyralspite” and “(u)grandite”, which were considered to represent two independent solid-solution series, were introduced by N.H. Winchell and A.N. Winchell (1927) in their well-known book “Elements of Optical Mineralogy”. Critical comments on the assumptions behind the classification scheme have been pointed out for at least 50 years, but it remains in use. There is more, though, behind this garnet classification scheme than just simple terminology. There are a long series of scientific discoveries and advances that are largely forgotten by the broader mineralogical community. They begin, here, with the work of the “father of crystallography”, René-Just Haüy, concerning the microscopic nature of crystals around 1780 and include later discoveries and proposals by Mitscherlich, Beudant, Wollaston and Koop relating to isomorphism and solid-solution behavior all before 1850. A second key era started with the discovery of X-ray diffraction in 1912 that allowed the atomic structures of crystals and furthermore atomic and ion radii to be determined. In terms of isomorphism and solid solution, the proposals and studies of Vegard, Zambonini, Wherry, A.N. Winchell and the “father of crystal chemistry” Goldschmidt are briefly discussed. The recognition of the sizes of atoms and ions, along with an understanding of chemical bonding behavior in crystals, was critical in the establishment of what can be termed “modern mineralogy”, a quantitative science as it is largely understood today that emerged by roughly the mid 1930s. The silicate garnet system pyrope-almandine-spessartine-grossular-andradite-uvarovite shows extensive homovalent substitutional solid solution over two structural sites and complete compositional variation between “pyralspite species” and “ugrandite species” has been documented. Thus, the prerequisites behind the terms “pyralspite” and “(u)grandite”, as originally formulated and often accepted even today, are incorrect and use of this classification is not recommended. Diffraction determinations of the volumes of garnet end members and volumes of mixing of garnet solid solutions give physical insight into solid-solution behavior. Today, investigations of local structural and crystal-chemical properties, together with determinations of lattice strain and thermodynamic mixing properties of silicate solid solutions, are leading to an ever more quantitative understanding of mineral behavior from the microscopic to macroscopic level.

71 **Key words:** Garnet, Mineral Classification, X-ray Diffraction, Crystal Structures,  
72 Atomic Theory, Isomorphism, Substitutional Solid Solution, Lattice Strain, Volume

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## INTRODUCTION

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76 Mineralogists, as a group, are quite zealous regarding terminology and classification. Since the  
77 beginnings of mineralogy a large number of articles, monographs, and textbooks have been  
78 devoted to the classification of minerals. Indeed, science begins with observation and description  
79 of the natural world and classification follows. In terms of the common rock-forming silicate  
80 garnets, general formula  $\{X_3\}[Y_2](Z_3)O_{12}$ , where dodecahedral  $\{X\}$ , octahedral  $[Y]$ , and  
81 tetrahedral  $(Z)$  represent the three special crystallographic cation sites and their polyhedral  
82 coordination in space group  $Ia\bar{3}d^1$ . One learns of the pyrospite and (u)grandite garnets and their  
83 solid-solution behavior. The two names were introduced by the father and son team of Newton  
84 Horace Winchell (1839-1914) and Alexander Newton Winchell (1874-1958) in their well-known  
85 book “Elements of Optical Mineralogy” (1927 – first edition 1909) and repeated in its later  
86 editions. They wrote, “If a mineral species is a natural substance which varies between definite  
87 natural limits in chemical composition then there are probably two and probably only two  
88 species in the garnet group, since three of the types commonly recognized belong to one  
89 continuous series and the other three to another. Neither species has received a name, but  
90 designations may be coined from the abbreviations of the names of the chemical species, as  
91 follows:

92

93		Pyrope	$Mg_3Al_2Si_3O_{12}$
94	Pyrospite . . .	Almandite	$Fe_3Al_2Si_3O_{12}$
95		Spessartite	$Mn_3Al_2Si_3O_{12}$

96

97

98		Uvarovite	$Ca_3Cr_2Si_3O_{12}$
99	Ugrandite . . .	Grossularite	$Ca_3Al_2Si_3O_{12}$
100		Andradite	$Ca_3Fe_2Si_3O_{12}$ ”.

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102 With time, as often is the case, terminology evolves and changes and it often becomes  
103 less precise. One reads in mineralogy textbooks of the two sets or sub-groups, namely the Ca

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<sup>1</sup> The use of the general formula  $E_3G_2Si_3O_{12}$  is recommended by Smith et al. (1998).



138 ***Molécules Intégrantes, Early Atomic Theory, Isomorphism, and Substitutional Solid***  
139 ***Solutions: The Birth of Mineralogy as a Science***

140 The history of mineralogy can, in a simple sense, be divided (see Schneer, 1995 and  
141 Hazen, 1984 for a more complete treatment) into two parts and is taken to start, here, in the late  
142 1700s. It begins with the concepts of *molécules intégrantes*, atomic theory, isomorphism, and the  
143 phenomenon of solid solution and how they started, evolved and were interrelated in study<sup>2</sup>.  
144 Treatments of the history of crystallography and the theories of crystal structures are numerous  
145 and each emphasizes certain scientific viewpoints or results over others or the contributions of  
146 one scientist over another, but the general interpretation is similar. Two books that discuss the  
147 subject at length, starting from the earliest times, include Burke (1966) and Authier (2013). The  
148 history of crystallography in its entirety is reviewed in a number of separate articles in Lima-de-  
149 Faria (1990) and chemical crystallography prior to the discovery of the diffraction experiment by  
150 Molcanov and Stilinovic (2014). Some of the major discoveries and theories, especially relating  
151 to the history of atomic theory, isomorphism and solid solution, which are discussed in short  
152 form here, are taken from the first two sources. In addition, a number of key original and review  
153 publications are presented.

154 The concepts of *molécules intégrantes* (integrant molecules), atomic theory, isomorphism,  
155 and solid-solution behavior were developed just following the age of enlightenment in Europe  
156 starting in the late 1700s and research followed into the first part of 1800s. During this period, a  
157 diverse range of scientists was trying to interpret and understand in a microscopic sense the  
158 nature of gases and liquids and, of interest here, crystals. Some investigators thought that crystals  
159 were made up of tiny microscopic second-order particles that were of identical shape and  
160 chemistry, but the exact nature of these particles was not fully clear. Here, the views of the  
161 prolific natural scientist, botanist, mineralogist and “father of crystallography”, the Frenchman  
162 René-Just Haüy (1743-1822), reigned supreme. He argued in 1792 that crystals, reflecting their  
163 consistent forms and obeying the crystallographic law of rational intercepts, consisted of so-  
164 called integrant molecules. In a physical sense, these integrant molecules could be considered as  
165 identical juxtaposed microscopic polyhedra that serve as the very tiny building blocks of crystals  
166 (see Fig. 2 for the case of garnet, which was the subject of Haüy’s first mineralogical memoir in  
167 1782). They were “the smallest corpuscles which would be obtained if mechanical division was  
168 pushed to its ultimate limit, had we sufficiently sharp tools” (Authier, 2013) and each crystal  
169 form had its own characteristic integrant molecule. It followed, in other words, from Haüy’s

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<sup>2</sup> From very early on the fields of chemistry and mineralogy were closely linked. Medieval and later scientists were often trained in theology and medicine and they studied the four sciences of the Quadrivium: arithmetic, geometry, astronomy and harmony (Schneer, 1995).

170 theory that “every chemical substance possesses a characteristic crystalline form, and that  
171 substances differing in chemical composition cannot occur in the same form” (Kraus, 1918).  
172 Haüy’s view dominated the field for a number of years and it appeared to explain well the  
173 observed variety of crystal forms and the law of rational intercepts (e.g., Burke, 1966). His work  
174 was clearly of benchmark importance for the fields of crystallography and mineralogy because it  
175 set out the idea of a space lattice for the first time. However, in spite of the theory’s  
176 attractiveness, contradictions and doubts on its correctness began to arise with further careful  
177 measurements on crystal forms and the angles between faces, as well as through the study of the  
178 chemistry of synthetic crystals and minerals. Here, scientists such as William Hyde Wollaston  
179 (1766-1828), François Sulpice Beudant (1787-1850) and Eilhardt Mitscherlich (1794-1863)  
180 made important contributions and discoveries (Burke, 1966).

181 Beudant, a former student of Haüy, researched (1817) the crystal forms of iron, copper  
182 and zinc sulfates crystallizing from solution. Haüy had proposed that these three different  
183 sulfates consisted of different integrant molecules namely, an acute rhombohedron, an irregular  
184 oblique-angled parallelepiped and a regular octahedron, respectively. Beudant wanted to  
185 investigate how much of another chemical component copper sulfate could accept without  
186 changing its crystalline form, for example. He found, surprisingly, that crystals containing 90%  
187 copper sulfate and only about 10 % iron sulfate yielded crystals having the same external  
188 rhombohedral form as pure iron sulfate. This finding, as well as other similar results for various  
189 crystal systems by other workers, was difficult to interpret using integrant molecules and these  
190 results produced heated debate among scientists throughout Europe (Burke, 1966).

191 It was the discovery and explanation, though, of isomorphism by the German chemist  
192 Mitscherlich that ultimately doomed Haüy’s crystal theory<sup>3</sup>. What are isomorphs and what is  
193 isomorphism<sup>4</sup>? The latter is a concept that for many years played an important role in  
194 mineralogy, though rather confused and variously defined (e.g., McConnell, 1943; Whittaker,  
195 1981; Jaffe, 1988). The term isomorphism today is largely unused and forgotten and mostly  
196 relegated to older mineralogical and crystal chemistry textbooks. According to Mitscherlich  
197 (1819; 1821), “An equal number of atoms, if they are bound in the same way, produce similar

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<sup>3</sup> See, for example, “Slaying the crystal homunculus” (Cahn, 1999) as well as Burke (1966) and Melhado (1980). The discovery of dimorphism and polymorphism by Mitscherlich in his studies on calcite and aragonite as well as sulfur crystals also contributed to the rejection of the theory of *molécules intégrantes*. Morrow (1969), taking a dissenting minority opinion, argued that W.H. Wollaston (see below) discovered isomorphism. Goodman (1969) discusses the views toward the atomic theory of Dalton in the 1800s and on how Wollaston vacillated in his interpretations on it.

<sup>4</sup> Different garnet species represent, with their well-pronounced {110} faces, textbook examples of isomorphs and garnet can form an isomorphic series in the classic sense meant by Mitscherlich (Fig. 3).

198 crystal forms, and the crystal form depends not on the nature of the atoms but on the number and  
199 method of combinations” (translated in Authier, 2013, page 331) or Mitscherlich’s last definition,  
200 “Substances possessing an analogous composition, which crystallize in the same form (or in  
201 similar forms) and which are capable of mixing in all proportions, are isomorphous” (Morrow,  
202 1969). And here it must be remembered that the precise physical nature of an atom prior to about  
203 1900 was not known. John Dalton (1766-1844), often considered to be the “father of atomic  
204 theory”, first presented his generalized ideas around 1803 based on his research on gases and  
205 liquid solutions (e.g., Dalton, 1803a,b). Mitscherlich accepted Dalton’s theory, but “professed  
206 ignorance of the shape or constitution of these atoms” (Burke, 1966).

207         The discovery of isomorphism<sup>5</sup>, which was based on Mitscherlich’s study of the  
208 compounds  $\text{KH}_2\text{PO}_4$  and  $(\text{NH}_4)_2\text{H}_2\text{PO}_4$  as well as  $\text{KH}_2\text{AsO}_4$  and  $(\text{NH}_4)_2\text{H}_2\text{AsO}_4$ , was of great  
209 consequence because isomorphous species can form substitutional solid solutions or as they are  
210 sometimes called mixed crystals (from the German *Mischkristalle*). In fact, the terms  
211 isomorphism and (substitutional) solid solution were used interchangeably for many years and  
212 among mineralogists the latter is used today<sup>6</sup>. The phenomenon of solid solution was difficult to  
213 explain using Häuy’s theory<sup>7</sup>. Morrow (1969) traces the discovery or observation of mixed  
214 crystals back to 1772 starting with the work of Rome De L’Isle. According to Burke (1966),  
215 Beudant, with his investigations such as the one noted above, and Wollaston should be given  
216 credit for the recognition of solid solution in crystals. W.H. Wollaston, an English medical  
217 doctor turned natural scientist, made thought provoking very early proposals on the microscopic  
218 nature of crystals. It was known that it was not possible to fill space completely with a  
219 tetrahedron or octahedron, for example, in the sense of Häuy’s theory. Thus, Wollaston wrote,  
220 “all difficulty is removed by supposing the elementary particles to be perfect spheres, which by  
221 mutual attraction have assumed that arrangement which brings them as near to each other as  
222 possible” (Wollaston, 1813, p. 54). He even constructed polyhedral models of various forms  
223 based on a close packing of hard spheres (Fig. 4), including some with two different types of  
224 spheres (i.e., alloy or solid solution).

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<sup>5</sup> Häuy wrote to Brochant, “if Mitscherlich’s theory is correct, mineralogy would be the most wretched of the sciences” (Kraus, 1918). However, as noted by Kraus, further careful work after Mitscherlich’s proposal, using more precise goniometers, showed that there are small differences in the angles between faces of isomorphous crystals of differing chemistry. “It was recognized rather early that the original idea of absolute identity of form, expressed by Mitscherlich, must be modified, absolute identity giving way to striking similarity of form.” (Kraus, 1918). Thus, Häuy’s pronouncements were in a certain sense partly correct.

<sup>6</sup> The crystallographic community recommended some time ago against using the term isomorphy and, thus, by extension isomorphism (see Lima de Faria et al., 1990).

<sup>7</sup> Häuy recognized that isomorphs and isomorphism occurred, but his interpretation of the phenomenon was incorrect (see Burke, 1966, or Melhado, 1980, for example).

225 The debate on the theory of isomorphism, both pro and con, following Mitscherlich's  
226 investigations was intense and much was at stake. For example, the well-known polymath,  
227 scientist, priest ([wikipedia.org/wiki/William\\_Whewell](http://wikipedia.org/wiki/William_Whewell)) and professor of mineralogy at  
228 Cambridge University (i.e., Trinity College), William Whewell (1794-1866), waded into the  
229 debate in a publication from 1831. He argued in favor of the theory of isomorphism partly based  
230 on his analysis of published garnet compositions and stoichiometric arguments. Although it  
231 appears that his proposed garnet stoichiometry was not fully correct, his analysis foresaw the  
232 nature of solid solution in silicate garnet in terms of the major cation substitutions at the X- and  
233 Y-sites. The problem was to obtain a stoichiometric formula based on 'wet' chemical analyses,  
234 which were sometimes rather inexact, from a solid-solution composition garnet. Whewell came  
235 to the conclusion that the divalent cations  $\text{Fe}^{2+}$ ,  $\text{Mn}^{2+}$ , Ca and Mg (X-site) belong together as do  
236 Al and  $\text{Fe}^{3+}$  (Y-site) in the garnet crystal-chemical formula.<sup>8</sup>

237 Following this often controversial but rich period of crystallographic research, further  
238 major developments with regard to the understanding of the internal structures of minerals was  
239 rather limited for a long time. It is important to note that the concept of single spherical atoms in  
240 crystals was pushed aside in many chemical and mineralogical studies. Research focused more  
241 on studying the masses and valences of elements and a more molecular approach or view, as  
242 starting largely from Haüy's work, was connected with crystals (Burke, 1966, page 124).

243 There was work done to better understand the phenomenon of isomorphism and the  
244 nature of "atomic" volumes in substances. Prior to the discovery of X-ray diffraction in 1912,  
245 neither a unit-cell shape and its volume nor the size of an atom in a modern sense could be  
246 measured directly. There was, though, another experimental means to determine "atomic"  
247 volumes. Here, the work of the German chemist H.F.M. Kopp (1817-1892), perhaps best known  
248 for the Kopp-Neumann rule governing heat capacity behavior, was important and he contributed  
249 to a further understanding of isomorphism and/or solid-solution behavior. His proposals, indeed,  
250 turned out to be quite foretelling (see later discussion), because he argued that similar or equal  
251 "atomic" (or smallest particle) volumes in crystals was a prerequisite for isomorphism<sup>9</sup>. He came  
252 to this conclusion through extensive measurements and analysis of a large number of isomorphic  
253 substances in which their "atomic volumes" were obtained by dividing their "atomic weights" by  
254 their specific gravity (Kopp, 1840; 1841). Otto (1848 - p. 126) wrote of "Kopp's Law"

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<sup>8</sup> As a matter of note, the general problem of assigning cations to certain crystallographic or structural sites from a chemical analysis, maintaining stoichiometry, and in calculating various end-member garnet components for a solid solution still exists today (see Locock, 2008).

<sup>9</sup> "Isomorphe Körper haben gleiches Atomvolum" und "Die kleinsten Theilchen isomorpher Körper sind nicht nur in der Form (Mitscherlich's Entdeckung), sondern auch in der Größe einander gleich" (Kopp, 1840, p. 3).

255 concerning the similar sizes of chemical atoms in isomorphous substances. The most renowned  
256 American mineralogist of the 19<sup>th</sup> century, James D. Dana (1813-1895), following upon the  
257 results and analysis of Kopp, also studied isomorphism and the nature of “atomic volume” for  
258 various minerals (Dana, 1850).

259         The situation in both chemistry and mineralogy was still quite confused, though, with  
260 regard to the nature and precise meaning of chemical formulae and atomic and molecular  
261 weights. The terms atoms and molecules were being used interchangeably, along with other  
262 expressions for the tiny particles thought to compose matter. For the sake of brevity, suffice it to  
263 state that the proposals of Stanislao Cannizzaro (1826-1910), an Italian chemist, at the world’s  
264 first international conference of chemistry at Karlsruhe, Germany in 1860<sup>10</sup>, were quite  
265 noteworthy. Cannizzaro’s research, using the earlier hypothesis of another Italian, Amedeo  
266 Avogadro (1776-1856), who worked on gases, led to a better and more precise understanding of  
267 the difference between atomic and molecular weights and how chemical formulae and reaction  
268 stoichiometries are to be expressed. It was an important step towards the development of modern  
269 atomic theory as understood today. In terms of minerals, the concept of integrant molecules was  
270 further weakened, although it must be noted, here, there does exist the very important class of  
271 molecular crystals especially in field of chemistry and the question of crystal structures was still  
272 open into the early 20<sup>th</sup> century. After this key conference at Karlsruhe, an internationally agreed  
273 upon table of modern atomic weights was adopted.

274         A further development regarding the possible nature of crystal structures and isomorphism  
275 can attributed to William Barlow (1845-1934), a geologist with an interest in crystallography (he  
276 derived the 230 space groups in 1894 slightly after E.S. Fedorov and A.M. Schoenflies), and  
277 William Jackson Pope (1870-1939), a chemist and crystallographer at Cambridge University.  
278 Several papers, both singled authored (i.e., Barlow) and together, were published on atomic  
279 theory, composition and crystal structures and one work from 1906 is especially significant. In  
280 this manuscript Barlow and Pope analyzed the possible structure of various crystals, mostly  
281 organic but also a couple of silicates, using models consisting of closest packed hard spheres.  
282 They wrote, “A crystal is the homogeneous structure derived by the symmetrical arrangement in  
283 space of an independently large number of spheres of atomic influence” and “a homogeneous  
284 structure or assemblage is one in which every point or unit possesses an environment identical  
285 with that of an infinitely large number of other similar points or units in the assemblage if the

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<sup>10</sup> See deMilt (1951).

286 latter is regarded as indefinitely extended throughout space.”<sup>11</sup> Later in the manuscript in terms  
287 of isomorphism they wrote, “It would seem that for two elements to be isomorphously  
288 replaceable one by the other, their spheres of atomic influence must be much more nearly of the  
289 same magnitude than if they are merely to possess the same valency.” Barlow and Pope  
290 discussed briefly the plagioclase feldspars as an isomorphous mixture stating, “They (albite,  
291  $\text{NaAlSi}_3\text{O}_8$ , and anorthite,  $\text{CaAl}_2\text{Si}_2\text{O}_8$ , – C.A.G.) have, however, the same valency volume,  
292 namely, 32, and by removing from the albite assemblage the group  $\text{NaSi}$ , of valency volume 5, it  
293 can be replaced without remarshalling, and indeed with but little disturbance of the crystalline  
294 structure, by the group  $\text{CaAl}$ , of the same valency volume.”

295 The various pre-diffraction proposals on atomic size and volume and their importance set  
296 the stage for more quantitative investigations in the 20<sup>th</sup> century.

297

## 298 **Modern Atomic Theory, Crystal Structures, Isomorphism, and Substitutional Solid**

### 299 **Solutions: The Beginnings of Modern Mineralogy**

300 It took time for the modern atomic revolution to come, but when it came it hit like a  
301 tsunami. In terms of crystals and their internal structures, the breakthrough was the discovery of  
302 X-ray diffraction in 1912 by M. Laue (1879-1960) and colleagues and crystal structure analysis  
303 by another father and son team, namely W.H. Bragg (1862-1942) and W.L. Bragg (1890-1971),  
304 as discussed recently in Eckert (2012) and Authier (2013). The Braggs, especially the son, using  
305 the new method soon after 1912, determined the crystal structures of a number of minerals with  
306 simple structures such as diamond, halite and sphalerite ( $\text{ZnS}$ )<sup>12</sup>. These discoveries and  
307 investigations and the scientific developments arising from the X-ray diffraction experiment can  
308 be considered as defining the second part of the history discussed here. It can be argued that they  
309 marked the beginning of modern mineralogy<sup>13</sup>, and it was physicists who made the breakthrough.  
310 In addition to the discoveries of diffraction and crystal structure analysis at roughly the same  
311 time, the fundamental physical concept of an atom came to light with the Rutherford-Bohr model.

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<sup>11</sup> Barlow and Pope, as other scientists before them and at the same time, did not understand the forces that acted on the atoms. They did propose the existence of both attractive and repellent forces acting between atoms, but ascribed them to gravity and kinetic energy, respectively.

<sup>12</sup> Gibbs et al. (2014) discuss the still widespread acceptance of the molecular crystal model for Earth materials at this time even after the determination of these simple structures and notably  $\text{NaCl}$  rock salt.

<sup>13</sup> The term “modern” obviously changes with time and some mineralogists soon recognized the revolutionary importance of the new scientific advances in the study of matter. See, for example, three presidential addresses to the Mineralogical Society of America entitled “The modern study of minerals” (Washington, 1925), “Isomorphous substitution of elements in minerals” (Phillips, 1932) and “The new mineralogy” (Winchell, 1933).

312 Unit-cell parameters can be measured via diffraction and the molar volume of a  
313 crystalline phase can be determined from the relationship:

314

$$315 \quad V = \frac{N_o}{Z} \left\{ \vec{a} \cdot \left[ \vec{b} \times \vec{c} \right] \right\} \quad (1),$$

316

317 where  $N_o$  is Avogadro's number ( $6.023 \times 10^{23}$  mole<sup>-1</sup>),  $Z$  is the number of formula units in the

318 unit cell and  $\vec{a}$ ,  $\vec{b}$  and  $\vec{c}$  are the lattice vectors. The scalar triple product of the lattice vectors

319 determines the volume of the unit cell. One also has the relationship:

320

$$321 \quad V = \frac{ZM}{N_o\rho} \quad (2),$$

322

323  $M$  is the molecular weight (g/mol) of the atoms in the unit cell and  $\rho$  is density (g/cm<sup>3</sup>).

324 Moore (1990) wrote that (2) is "Perhaps the earliest known and most important relationship

325 between the underlying parameters for any crystalline substance....". He also states it neatly

326 divides chemical crystallography into the pre- and post-diffractions eras. When determinations

327 for the different parameters are available, this equation can be used as a check for their

328 correctness. It has, for example, been used to test if analytical measurements of composition of a

329 crystalline phase, which can be rather complex, are complete and correct.

330 Not long after 1912 many different scientists were undertaking diffraction investigations

331 and measurements were even made on solid solutions. A notable example for the latter was the

332 work of Vegard (1921). He studied several simple binary solid solutions including the system

333 KCl-KBr. His powder diffraction measurements showed that the reflections for the

334  $K(Cl_{0.50}Br_{0.50})$  composition were sharp and that their positions were located between those of

335 end-member KCl and KBr (Fig. 5). This demonstrated, first, that the crystal involved "direct

336 atomic substitution" and that it was not a physical mixture of two intergrown phases. Second, he

337 showed that the unit-cell edge,  $a$ , and the "molecular volume" along the KCl-KBr binary obeyed

338 the "law of additivity", that is, they varied in a linear and continuous fashion. The expression

339 defining, what is now termed Vegard's law, the unit-cell parameter for a binary solid solution

340 AB is:

341

$$342 \quad a_{A_{(1-X_B)}B_{X_B}} = a^{\circ}_A(1 - X_B) + a^{\circ}_B(X_B) \quad (3),$$

343

344 where  $X_B$  is the mole fraction of component B and  $a^\circ_A$  and  $a^\circ_B$  are the unit-cell parameters (or  
345 lattice parameters) of the two end-member components A and B, respectively.

346 It is difficult from today's more "enlightened" understanding to fully grasp the cloud of  
347 uncertainty in which mineralogists (and others as well) considered the important class of silicates  
348 during this period. This is reflected in the 1922 article of the well-known physicist, physical  
349 chemist and metallurgist G. Tammann (1861-1938) entitled "On the Constitution Question of  
350 Silicates". His article introduces and addresses the question of whether silicates could be  
351 molecular in structure analogously to the large group of organic carbon-based compounds! Of  
352 course, the nature of chemical bonding in crystals and certainly silicates was unknown and had  
353 to wait a few years for a full and correct interpretation (e.g., Pauling, 1929)<sup>14</sup>. Interestingly,  
354 Tammann ultimately concluded, using simple specific heat capacity data on several silicates, as  
355 well diffusional behavior of various compounds and the known crystallization behavior of some  
356 silicates from melts, that a transfer of molecular theory from organic chemistry to the silicates  
357 was not valid.

358 A few classically trained mineralogists did soon, though, understand the link among  
359 post-1900 atomic theory, crystal structures, and isomorphism. Wherry (1923) discussed the key  
360 role that atomic volume plays in determining isomorphism in various silicates. He wrote, "It has  
361 long been held that in order to be able to replace one another, elements must be chemically  
362 analogous and of equal valence" and then followed with "It now seems more probable that the  
363 principle requisite of isomorphous replaceability is that the elements in question must possess  
364 approximately identical volumes, at least in simple compounds, the crystal structures of which  
365 represent fairly close packing of the constituent atoms". Similar views on isomorphism were also  
366 published by Zambonini (1922) and Zambonini and Washington (1923), where emphasis was  
367 placed on heterovalent isomorphism, i.e., [Na,Si]-[Ca,Al] exchange in plagioclase. These  
368 proposals and interpretations, though beset by only a rough knowledge of atomic sizes and/or  
369 volumes, a great uncertainty of the nature of silicate crystal structures, and in chemical bonding<sup>15</sup>,  
370 were new. The situation was far from certain, though, with the acknowledged crystal chemist  
371 and crystallographer R.W.G. Wyckoff (1923) ending his exposition on isomorphism with  
372 "Enough has been said to show that too few data are at hand to give an adequate explanation of  
373 isomorphous mixing, even in the relatively simple example of albite and anorthite."

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<sup>14</sup> See Gibbs et al. (2014) for a recent discussion of bonding behavior and ionic radii in minerals and a historical summary of their development.

<sup>15</sup> Wherry (1923), harking back to the question put forward by Tammann (1922), noted that the lack of isomorphism between carbonates and silicates is a result of the volume (size) difference between carbon and silicon!

374 Another classical mineralogist who recognized the importance of the property of atomic  
375 size in terms of isomorphism turns out to be A.N. Winchell, who is primarily known for his  
376 extensive work and books on the optical properties of minerals (N.H. Winchell and A.N.  
377 Winchell, 1927, and later editions). Winchell published an article in 1925 in the journal *Science*  
378 entitled “Atoms and Isomorphism” in which he presented his ideas and analysis.<sup>16</sup> Winchell  
379 began his article with the statement, “Atoms were formerly known only by their weights and  
380 chemical properties. . . . I shall try to show that one of the properties of atoms depends upon their  
381 sizes rather than their weights.”<sup>17</sup> He listed four isomorphous mineral groups namely: I. CaCO<sub>3</sub>,  
382 MgCO<sub>3</sub>, FeCO<sub>3</sub>, MnCO<sub>3</sub>, and ZnCO<sub>3</sub>, II. CaSO<sub>4</sub>, SrSO<sub>4</sub>, BaSO<sub>4</sub>, and PbSO<sub>4</sub>, III. MgFe<sub>2</sub>O<sub>4</sub>,  
383 FeFe<sub>2</sub>O<sub>4</sub>, ZnFe<sub>2</sub>O<sub>4</sub>, and NiFe<sub>2</sub>O<sub>4</sub>, and IV. MgFe<sub>2</sub>O<sub>4</sub>, MgAl<sub>2</sub>O<sub>4</sub>, and MgCr<sub>2</sub>O<sub>4</sub> and discussed the  
384 possible nature of the exchange among cations of the same valence.

385 In further more detailed discussion, he used the structure of NaCl as a model to discuss  
386 atomic substitution in crystals. Winchell wrote, “If any atoms other than those of Na or Cl exist  
387 in a NaCl crystal (not merely mechanically enclosed) they must either replace some of the atoms  
388 of the NaCl space lattice, or be small enough to find places between these, as very fine sand can  
389 find places between the grains of very coarse sand, even though the latter are in contact. Both  
390 these cases probably occur in crystals, but it is plainly only the first case which can lead to an  
391 isomorphous series”. He wrote, further, “With a gradual increase in the relative number of Br  
392 atoms a series can be imagined extending from pure NaCl to pure NaBr.” He also wrote, “Now,  
393 if crystals are close-packed space lattices built out of atoms, and if isomorphous systems can be  
394 formed only by the replacement in the space lattice of one kind of atom by another, it is evident  
395 that the size (or domain) of the atoms must be very important in determining what atoms can  
396 mutually replace one another in such systems. This principle, that atoms must be of nearly the  
397 same size in order to be able to form isomorphous systems in various compounds, seems far  
398 more important than the old idea that the atoms must be of the same valence.”

399 Winchell also discussed the two silicate garnet species, which at this time he termed the  
400 “two garnet systems”. He wrote, “The members of each system are mutually miscible (in  
401 crystals) in all proportions, since Al, Fe and Cr are similar in size, and also Mg, Fe and Mn, but  
402 the members of one system show only partial miscibility with members of the other system,  
403 since Ca has nearly twice the volume of Mg, Fe or Mn.”

404 It is exactly these views on atomic mixing and isomorphism that were put on a more  
405 quantitative and extended basis by the work of the well-known geochemist and mineralogist

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<sup>16</sup> The “Web of Science” shows zero citations.

<sup>17</sup> Winchell, as Wherry (1923), uses “atomic radii” (see W.L. Bragg, 1920). Landé (1920) published a short list of ionic radii for the alkali metals and halogens.

406 Victor Moritz Goldschmidt (1888-1947)<sup>18</sup>. Goldschmidt (1926a, b), together with the help of co-  
407 workers, published a list of ionic radii and he laid out his ideas on atomic exchange in crystals.  
408 Goldschmidt and his co-workers were able to determine the ionic radii of various metals by  
409 determining the unit-cell dimensions of various AX and AX<sub>2</sub> oxide and fluorine compounds,  
410 using the results of Wasastjerna, published in 1923, who determined the ionic radii of F<sup>-</sup> (1.33 Å)  
411 and O<sup>2-</sup> (1.32 Å) anions from optical considerations (see Mason, 1992). To summarize briefly,  
412 for example, Goldschmidt stated that extensive miscibility in minerals could occur below the  
413 melting point when the difference between the ionic radii of the exchangeable “atoms”<sup>19</sup> is not  
414 greater than 15%. Based on this extensive and careful research, Goldschmidt is considered one  
415 of the “fathers of modern crystal chemistry” (Mason, 1992).

416         Returning to the slightly earlier paper of Winchell (1925), he also addressed the role of  
417 temperature in affecting solid-solution behavior, for example with the alkali feldspars NaAlSi<sub>3</sub>O<sub>8</sub>  
418 and KAlSi<sub>3</sub>O<sub>8</sub> and he wrote, “It is an interesting fact that, in this case and in some others, two  
419 substances, whose unlike atoms differ so much in size that the isomorphism is only partial at  
420 ordinary temperature, exhibit perfect isomorphism at high temperature, as if the expansion of the  
421 space lattice due to heat were sufficient to permit free replacement of the small atoms by larger  
422 ones at high temperature, even though that is impossible at low temperatures.” The sizes of  
423 atoms and ions are common knowledge today, as taught in introductory chemistry and  
424 mineralogy courses, but this was not the case in the mid-1920s. Winchell understood early on the  
425 relationships between crystal form, structure, composition and the nature of solid solution on an  
426 atomic basis. The importance of solid solution in many minerals is further reflected in his  
427 extensive investigations of their optical properties (i.e., *Elements of Optical Mineralogy*, 1927,  
428 and later editions). The early proposals of Wherry, Zambonini and Winchell, and especially the  
429 extensive investigations and analysis of Goldschmidt and coworkers, were an essential  
430 development in mineralogy because they extended the interest and emphasis of atoms in  
431 minerals and crystals beyond just their masses and charges to their sizes (radii), as had been done  
432 for many years following the discoveries of Mitscherlich.

433         In the second half of the 1920s the study of crystals at an atomic level was moving fast,  
434 physical understanding was increasing greatly and the first silicate structures were determined.  
435 Interestingly, in completing the circle in terms of a common theme of this paper it turns out,  
436 once again, that garnet is fundamental. In 1925 the crystallographer and mineralogist G. Menzer

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<sup>18</sup> Neither Winchell nor Goldschmidt, for example, adopt the term solid solution, but instead use isomorphism to denote it. Goldschmidt wrote, “Als Isomorphie bezeichne ich im folgenden die Erscheinung, daß Substanzen analoger chemischer Formel Analogie der Krystallstruktur aufweisen.”

<sup>19</sup> Krystallbausteine.

437 (1897-1989)<sup>20</sup> determined, using the new technique of X-ray diffraction, the crystal structure of  
438 grossular. His study was one of the first correct determinations of a silicate structure<sup>21</sup>. Menzer  
439 followed in 1928 in a classic and extensive work, where he showed that the six common end-  
440 member silicate garnets (i.e., pyrope, almandine, spessartine, uvarovite, andradite, and grossular),  
441 which were known to be isomorphous, were also isostructural with one another. Modern  
442 mineralogy had taken its first steps.

443 By the middle of the 1930s the state of the field was largely established, as it is  
444 essentially known today. For example, W.L. Bragg (1930) reviewed the crystal structures of  
445 many of the rock-forming (alumino-)silicates, with the notable exception of the feldspars, that  
446 within the span of previous five years had been determined. Grimm and Wolff published in 1933  
447 a long and quantitative review article discussing the physical and chemical state of atoms and  
448 ions, the different types of bonding behavior, chemical complexes and the crystal chemistry of  
449 various crystalline phases.

450

## 451 **A TALE OF TWO GARNETS: COMPOSITIONAL RANGE OF THE COMMON AND** 452 **“NOT-SO-COMMON” SILICATE GARNETS**

### 453 **Natural Garnets**

454 The pyralspite and (u)grandite classification scheme of Winchell and Winchell (1927) is  
455 based on the earlier compilation and analysis of many garnet compositions of Boeke (1914).  
456 Figure 6, taken from the former authors, is slightly modified from Boeke (1914). The lower part  
457 of the figure shows the reported range of garnet compositions between grossular and andradite  
458 (grandite), while the cluster of data in the upper left shows the composition range of the  
459 pyralspite species.<sup>22</sup> Of particular relevance is the apparent lack of garnet compositions falling  
460 between the pyralspite and grandite fields, and hence, the classification of two different silicate  
461 garnets.

462 There is, of course, a multitude more of chemical analyses on garnet available today than  
463 in 1914 and the compositional range of natural silicate garnets has been researched and “mapped”  
464 out to great detail (e.g., Grew et al., 2013). What do the data say in terms of the range of  
465 compositions within the system pyrope-almandine-spessartine-grossular-andradite-uvarovite? To

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<sup>20</sup> The garnet menzerite-(Y), ideal end member  $\{Y_2Ca\}[Mg_2](Si_3)O_{12}$ , was named in his honor by Grew et al. (2010).

<sup>21</sup> The crystal structures of  $\alpha$  and  $\beta$  quartz were also published in 1925 by W.H. Bragg and R.E. Gibbs. W.L. Bragg and West (1927) wrote (p. 452), “It (i.e., Menzer’s analysis of the garnet structure - CAG) was the first analysis of a complex silicate to be made, and illustrates the characteristic features of these compounds in a very striking way.”

<sup>22</sup> The ferric-iron contents of many of the pyralspite compositions are too high and are due to problems with the older ‘wet’ chemical analyses.

466 give the simple and long-known answer first, there is compositional variation between the  
467 pyralspite and (u)grandite “species”. Extensive, if not complete, substitutional solid solution is  
468 observed between and/or among a number of the end-member garnet components. A number of  
469 examples of early, published garnet compositions, which by no means are intended to be  
470 complete, given by crustal metamorphic and higher-pressure upper mantle garnets are briefly  
471 discussed.

472 Lee (1958) described a largely andradite-spessartine-(grossular) garnet from Pajsberg,  
473 Sweden in a rock consisting of rhodonite, garnet and clinopyroxene. A recalculation of the  
474 crystal-chemical formula in terms of various garnet end members, using the calculation scheme  
475 of Locock (2008), is given in Table 1. Lee also described later a largely spessartine- grossular  
476 garnet from the Victory Mine at Gabbs, Nevada (Lee 1962). The garnet occurs in “the sheared  
477 and feldspathized portion of a granodiorite” and its composition is also listed in Table 1.  
478 Ackermann et al. (1972) described almandine-grossular-rich crystals, which are compositionally  
479 zoned, from epidote-bearing gneisses and mica schists from the Western Hohe Tauern, Austria.  
480 The garnets crystallized at the greenschist to low-temperature amphibolite facies and one sample  
481 composition (123 - core) is listed in Table 1. Indeed, there are a number of reports of garnet  
482 approaching roughly 50-50 mole % almandine-grossular composition in quartzofeldspathic  
483 gneisses (e.g., Ashworth and Evirgen, 1984 - rim composition of their garnet M.56.4 E is given  
484 in Table 1). All these crustal garnets show extensive solid solution between pyralspite and  
485 (u)grandite.

486 The validity of the pyralspite-(u)grandite classification is further contradicted by garnets  
487 from higher pressure rocks. Sobolev et al. (1973) describe several suites of chromium-bearing  
488 garnets sampled from kimberlites. These garnets show a range of Cr<sub>2</sub>O<sub>3</sub> concentrations and a few  
489 samples show compositions approximating the pyrope-uvarovite binary. Their sample S-1 has  
490 roughly 41 mole percent pyrope-almandine and 46 mole percent uvarovite (Table 1). O’Hara and  
491 Mercy (1966) noted a violation of the Winchells’ classification scheme in their study of “calcic  
492 pyralspites” found in kyanite eclogite xenoliths from the Roberts Victor kimberlite mine in  
493 South Africa. A garnet from rock sample 37077 has the composition of very roughly  
494 Py<sub>25</sub>Alm<sub>25</sub>Gr<sub>50</sub> (Table 1). Sobolev et al. (1968) described, in detail, a wide range of grossular-  
495 rich pyrope-almandine garnets in so-called grosopydite xenoliths from the Zagadochnaya  
496 kimberlite in Yakutia, Russia. Later, he and colleagues described more nearly binary grossular-  
497 pyrope garnets in high pressure diamond-bearing crustal carbonate-silicate rocks of the  
498 Kokchetav Massif in Kazakhstan (Sobolev et al., 2001). The core composition of these garnets is  
499 given in Table 1.

500 Finally, it goes without saying that ultra-high-pressure majoritic-bearing garnets (Smith  
501 and Mason 1970 - with majorite end member as  $\{\text{Mg}_3\}[\text{SiMg}](\text{Si}_3\text{O}_{12})$  - described long after the  
502 pyralspite-(u)grandite scheme was proposed, are not at all covered by the pyralspite-ugrandite  
503 classification scheme. Actually, it may be the case that many, possibly even the bulk of silicate  
504 garnet in the Earth is not covered by the scheme, because the transition zone is composed of  
505 approximately 40% majoritic garnet (e.g., Irifune, 1987)!

506

### 507 **Synthetic Silicate Garnets**

508 In addition to the analysis afforded by natural garnets, quantitative information on the  
509 possible compositional range of silicate garnet can be obtained by experimental laboratory  
510 investigations. Interestingly, Boeke (1914, p. 153) wrote more than 100 years ago that  
511 equilibrium investigations would “provide the final word” on this question. In the lab, unlike  
512 nature, the composition of a thermodynamic system can be chosen and  $P$  and  $T$  fixed exactly.  
513 The first laboratory test of Boeke’s assertion was only possible nearly 5 decades later in the late  
514 1950s following technical developments in high-pressure devices. The synthesis and phase  
515 relations of grossular, pyrope and almandine and their solid solutions, which are not stable at 1  
516 atm, could now be investigated. Chinner et al. (1960) were probably the first to show that  
517 extensive solid solution between pyrope and grossular, as well as between pyrope and almandine,  
518 was possible. Their first results on these two binaries were later confirmed a number of times in  
519 different investigations. In addition, a number of other binary garnet solid solutions have been  
520 synthesized in the lab including almandine-grossular, spessartine-grossular, spessartine-pyrope,  
521 spessartine-uvarovite, andradite-grossular, grossular-uvarovite, and almandine-spessartine (Fig.  
522 7). Of course, various ternary, quaternary and higher-order compositions within the six-  
523 component system pyrope-almandine-spessartine-grossular-andradite-uvarovite have been  
524 synthesized and their phase relations studied as well.

525

### 526 **Conclusion: The pyralspite-(u)grandite and other garnet classification schemes**

527 Geiger (2008) wrote, “The Pyralspite-Ugrandite classification scheme has outgrown its  
528 usefulness in terms of describing and understanding the silicate garnets in terms of their  
529 compositions and stabilities. ... It is concluded that the Pyralspite-Ugrandite classification for  
530 silicate garnets should largely be dropped - or at best be used for purely mnemonic purposes. It  
531 should not be used to interpret solid-solution behavior, the occurrence of certain garnet  
532 compositions in nature or the lack thereof, or to infer major differences in bonding and crystal-  
533 chemical behavior between the two groups.” This has been done in some studies on garnet.

534 Are other classifications better and/or are they even necessary and, if so, to what  
535 scientific extent? Sobolev (1964), following along the lines of Boeke (1914) some five decades  
536 later, in an analysis of a large number of natural garnet compositions, proposed a classification  
537 scheme different than that of the Winchells. He recognized four main species, almandine-pyrope,  
538 almandine-spessartite, spessartite-grossularite, and grossularite-andradite with a possible fifth  
539 species for titanium-bearing garnet. Sobolev's diagram for the ternary system almandine-pyrope-  
540 spessartine, for example, shows compositions of natural garnets essentially spanning the entire  
541 compositional range between pyrope and almandine and between almandine and spessartine, but  
542 with very limited compositional variation between spessartine and pyrope. Boeke (1914) noted  
543 for all intents and purposes the same. The ternary almandine-pyrope-spessartine in both works is  
544 characterized by a large compositional gap at the pyrope-spessartine binary that extends into the  
545 ternary system. This behavior is not correct however. Research starting around the late 1970s  
546 demonstrated that natural largely pyrope-spessartine garnets<sup>23</sup> (and nearly ternary pyrope-  
547 spessartine-almandine compositions), though rare, do occur in nature (Table 1). Interestingly,  
548 they are often referred to as color change garnets that are typically characterized by pink to  
549 pinkish orange colors (Schmetzer et al., 2001), but in a few instances they may have a blue-green  
550 (e.g., Schmetzer and Bernhardt, 1999) or even a deeper blue color in daylight<sup>24</sup>. Localities for  
551 these garnets are in East Africa (Umba mining region), Sri Lanka and Madagascar. There should  
552 be complete solid solution over the whole pyrope-almandine-spessartine ternary at high  
553 temperatures and pressures.

554 Summarizing, garnet compositions falling between the pyralspite and (u)grandite fields  
555 do occur. This conclusion was reached before (e.g., Sobolev, 1964; Nemeč 1967). The pyralspite  
556 and (u)grandite species or series and the like (various sources) have no validity. There is little  
557 scientific reason or justification for using any mineralogical classification for the common  
558 silicate garnets that is based on the reported degree of solid solution among or between the  
559 various end-members. In a detailed contribution on the question of garnet classification, Grew et  
560 al. (2013) list all the common silicate garnets, as well as others, into a single supergroup. The  
561 various garnet species names simply correspond to the dominant end-member component in a  
562 solid-solution crystal.

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564

## THE PRESENT STATE OF THE FIELD

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<sup>23</sup> Wherry (1923) noted that the absence of any natural pyrope-spessartine garnets at that time had a chemical and not a volume origin!

<sup>24</sup> With their discovery, the color of natural silicate garnet spans the entire color range of the visible light spectrum – violet, blue, (cyan), green, yellow, orange and red (also colorless or white, as well as black, garnets can be found).

565 **Substitutional Solid Solutions: Atomic Size and Volumes, Structural and Thermodynamic**  
566 **Properties, and the Role of Strain**

567 The structural and crystal-chemical properties of the common silicate garnets, both  
568 (nearly) end-member and solid-solution compositions, and the relationships among them have  
569 been analyzed by X-ray single-crystal diffraction methods (e.g., Novak and Gibbs, 1971;  
570 Armbruster et al., 1992; Merli et al., 1995) and using various spectroscopies (Geiger, 2004)  
571 many times. The experimental results show that the “pyralspites” and “ugrandites” are separated  
572 by certain characteristic structural behavior such as bond length (Fig. 8; see also Novak and  
573 Gibbs, 1971). The whole story does not, though, end here.

574 There are two points to be addressed. The first has to do with chemistry. The  
575 thermodynamic stability or occurrence of a phase with respect to another phase or phase  
576 assemblage is a function of temperature, pressure and composition of the system, as discussed  
577 above. Considering the enormous range of  $P$ - $T$  conditions existing in the Earth, one can conclude  
578 that the occurrence of any given silicate garnet is determined by the different bulk-composition  
579 systems that occur in nature. The relatively few number of garnet compositions falling between  
580 the pyralspites and (u)grandites (Figs. 6 and 8) reflects the lack of appropriate bulk compositions  
581 necessary for these garnets to crystallize. This simple fact is sometimes forgotten or overlooked.

582 The second point is structurally and thermodynamically related. It involves the important  
583 effect that the size of atoms has on solid-solution behavior, a main theme of this manuscript. The  
584 size of the mixing atoms is reflected in the macroscopic thermodynamic properties, for example,  
585 in the volume of mixing behavior for say a garnet solid solution. Volume can be measured with  
586 high precision and accuracy, provided good crystals can be synthesized. It often turns out that  
587 many solid solutions, including those of garnet, do not strictly obey Vegard’s law - eqn. (3).  
588 Unit-cell edges and molar volumes can show slight deviations from linearity across a binary (i.e.,  
589 ideality in a thermodynamic sense,  $\Delta V^{mix,ideal}$ ). This gives rise to an excess volume of mixing,  
590  $\Delta V^{excess}$ , as defined by

591

592 
$$\Delta V^{excess} = \Delta V^{mix,real} - \Delta V^{mix,ideal} = \Delta V^{mix,real} - \sum X_i V_i^o \quad (4),$$

593

594 where  $\Delta V^{mix,real}$  is the real or measured volume of mixing and  $V_i^o$  the volume of the different  
595 end-member phases. For silicate substitutional solid solutions with random or nearly random  
596 atomic mixing,  $\Delta V^{excess}$  often deviates from  $\Delta V^{mix,ideal}$  in a positive manner and is seldom  
597 negative (Geiger, 2001). When atomic ordering occurs,  $\Delta V^{excess}$  can be negative in behavior.

598  $\Delta V^{ex}$  behavior can be described to first order using a so-called symmetric mixing model,

599 where  $\Delta V^{excess} = W^V \cdot X_A(1 - X_A)$  and  $W^V$  is the volume interaction parameter and  $X_A$  the mole  
600 fraction of component A. Figure 9a shows excess volumes,  $\square\square\square\square\square\square\square\square\square\square\square$ , for six  
601 binary aluminosilicate (i.e.,  $X_3Al_2Z_3O_{12}$ ) garnet solid solutions as a function of the volume  
602 difference,  $\Delta V$ , where  $\Delta V = (V_B^o - V_A^o) / V_{BA}^-$  with  $V_B^o$  the molar volume of the larger component,  
603  $V_A^o$  that of the smaller component and  $V_{BA}^-$  the mean of the two.  $W^V$  is a positive function of  $\Delta V$ ,  
604 but the choice between linear and quadratic behavior is difficult to make. Computer simulations  
605 on hypothetical binary aluminosilicate garnet solid solutions, made with empirical pair potentials,  
606 show, however, that quadratic behavior is expected (Fig. 9b - Bosenick et al., 2001). It was also  
607 shown by these authors that the excess enthalpy of mixing,  $\Delta H^{excess}$ , behaved similarly (cf.,  
608 Davies and Navrotsky, 1983). This quadratic dependence of the  $\Delta^{excess}$  and  $\Delta H^{excess}$  for binary  
609 homovalent solid-solution systems is an important result with regard to understanding  
610 thermodynamic mixing behavior.

611 The recent experimental and computational modeling results build upon the earlier works  
612 from the 1910s to 1930s discussed above in terms of atomic size and solid-solution behavior and  
613 they are a quantitative extension of them. It is expected that size difference(s) among the mixing  
614 cation determines the thermodynamic behavior for solid solutions in the system pyrope-  
615 almandine-spessartine-grossular-uvarovite-andradite<sup>25</sup> (see Ganguly and Kennedy, 1974, for  
616 early work on garnet in this direction and for other solid-solution phases Davies and Navrotsky,  
617 1983). Table 2 lists the difference in molar volume,  $\Delta V_M$ , between various garnet end members  
618 for the 15 binaries of the six-component system (Fig. 7). Garnets for three binaries in Table 2  
619 have not been found in nature nor synthesized in the laboratory. It must be noted, though, that in  
620 comparing the various  $\Delta V_M$  values that different types of structural cation mixing in garnet  
621 occurs, involving either the X- and Y-site or both. The structures and local microscopic strain,  
622 and therefore thermodynamic mixing behavior, appear to respond differently depending upon  
623 which crystallographic site(s) solid solution takes place (Woodland et al. 2009). They argue that  
624 binaries involving the mixing of trivalent cations at the Y-site often, but not always, show  
625 negative  $\Delta V^{ex}$  behavior.

626 Consider further pyrope-grossular,  $\{Mg_{3-x}Ca_x\}Al_2Si_3O_{12}$ , garnets, because they have  
627 been intensively studied and their structural-property relationships are the best understood. This  
628 binary is characterized by substantial positive nonideality in all the macroscopic thermodynamic

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<sup>25</sup> Fyfe (1951) in his paper "Isomorphism and Bond Type" discussed the role of chemical bonding in affecting solid-solution behavior in various phases. Other physical factors can also play a role such as atomic valence, electronegativity differences, and electronic effects such as crystal field stabilization energies.

629 mixing functions  $\Delta G^{mix}$ ,  $\Delta H^{mix}$ ,  $\Delta S^{mix}$  and  $\Delta V^{mix}$  (Dachs and Geiger, 2006).  $\Delta H^{mix}$  behavior  
630 (Newton et al., 1977) is important in terms of stability, as this function is what largely controls  
631 the miscibility gap along the pyrope-grossular join (Fig. 10). Complete solid solution between  
632 pyrope and grossular is only achieved at high temperatures and pressures<sup>26</sup>, whereas there should  
633 be unmixing at lower temperatures for more pyrope-rich compositions (Dachs and Geiger  
634 2006).<sup>27</sup> What on a local scale is causing the destabilizing positive  $\Delta H^{mix}$  behavior, which should  
635 lead to exsolution of Ca-rich and Mg-rich garnets? The answer is microscopic lattice strain. This  
636 was measured using X-ray powder diffraction measurements on a series of synthetic  $\{Mg_xCa_{1-x}\}_3Al_2Si_3O_{12}$   
637 solid-solution garnets (Dapiaggi et al., 2005; see also Du et al., 2016). These high-  
638 resolution synchrotron measurements allow a quantitative determination of minor variations in  
639 powder reflection line widths that, in turn, give information on elastic strain. The results show  
640 that strain is smallest for end-member grossular and pyrope, which have no local structural  
641 heterogeneity caused by Ca and Mg mixing. Intermediate compositions, on the other hand, show  
642 reflection broadening reflecting their local structural heterogeneity. This produces elastic strain  
643 that is asymmetric in nature across the binary, which is similar in behavior as  $\Delta H^{mix}$  (Dapiaggi et  
644 al., 2005). The simplest crystal-chemical interpretation is that it is easier to incorporate a smaller  
645 Mg cation (0.89 Å – Shannon, 1976) in a larger volume grossular-rich host than a larger Ca (1.12  
646 Å) cation in a smaller volume pyrope-rich garnet. The actual physical situation at the local level  
647 is more complicated, though, because it involves the distortion of strongly bonded  $SiO_4$  and  $AlO_6$   
648 groups that are edged-shared to  $(Mg/Ca)O_8$  dodecahedra (see Bosenick et al., 2000). It is this  
649 distortion and/or stretching of strong bonds that significantly affects  $\Delta H^{mix}$ . It follows that, for  
650 the six aluminosilicate binary solid solutions (Fig. 7), pyrope-grossular garnets are the most  
651 nonideal (Bosenick et al. 2001).

652

## 653 **IMPLICATIONS AND FUTURE RESEARCH**

654

655 Much has been learned about the chemical and physical properties of minerals over the  
656 past 250 years, but the scientific quest is certainly far from over. Most rock-forming minerals,  
657 which are largely silicates, are substitutional solid solutions. Their macroscopic thermodynamic  
658 properties, and thus their stabilities in the Earth, are a function of complex local structural and  
659 crystal-chemical properties. Both the microscopic and macroscopic realms and the link between

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<sup>26</sup> It is interesting, furthermore, to note that complete solid solution between pyrope and grossular is possible although the difference in radii between Mg and Ca is greater than 15%.

<sup>27</sup> Reports of exsolution in natural garnet exist (e.g. Wang et al., 2000), but more definitive study is required, especially with regard to the crystallographic and microscopic aspects.

660 them are required to achieve a full understanding of solid-solution behavior. What is, briefly, the  
661 state of the research field today? A number of issues were already discussed in Geiger (2001).  
662 New understanding on microscopic structural properties is coming from a variety of different  
663 spectroscopic measurements, which are continually increasing in sophistication, and are also  
664 being made at different temperatures and pressures (for garnet see Geiger, 2004, and for a review  
665 of spectroscopic methods, in general, see Henderson et al., 2014). Diffraction- and  
666 spectroscopic-based results are highly complementary and go hand-in-hand in describing crystal  
667 properties over different length scales.

668         Local crystal-structure properties are also now being investigated computationally. There  
669 are a number of different approaches (*e.g.* static lattice energy with empirical pair potentials,  
670 quantum mechanical first principle, Monte Carlo, molecular dynamics, pair distribution function  
671 analysis) that allow ‘simulation experiments’ (Geiger 2001). Indeed, many experimental tools  
672 are decidedly blunt when it comes to investigating atomistic-level properties and computer  
673 simulations have opened up a whole new area of research. In some cases, even first principle  
674 calculations are now possible on relatively complex crystal structures with larger unit cells.  
675 However, the study of many key silicate solid solutions (*e.g.*, garnets, micas, amphiboles,  
676 pyroxenes), especially those containing transition metals, still remains a serious challenge.

677         In terms of theory, little is understood about the precise nature of local strain fields and  
678 the nature of their interactions and what their associated elastic energies are in a solid solution  
679 beyond minor-element substitution levels. At minor concentration levels of atomic substitution,  
680 physical models describing strain energy and element partitioning behavior, for example, have  
681 been formulated under simplified assumptions (*e.g.* Eshelby, 1954; 1955; Nagasawa, 1966; Brice,  
682 1975). However, constructing physical models to describe strain is very difficult for complex,  
683 low-symmetry anisotropic structures, as in many silicates. In addition to strain fields and elastic  
684 energies, electronic and magnetic behavior in solid solutions and their effect on macroscopic  
685 physical properties are poorly known. For example, it is not well understood: i) how bonding  
686 character (Gibbs et al., 2014) may vary slightly across a homovalent binary solid solution (see  
687 Geiger, 2008, for the case of aluminosilicate garnet), ii) how magnetic properties and phase  
688 transitions behave as a function of composition, iii) how magnons and phonons can interact and,  
689 thus, affect macroscopic thermodynamic behavior, and iv.) how electronic high spin-low spin  
690 transitions behave as a function of pressure, temperature and composition.

691

692

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693

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699

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- 939

## FIGURES

940

941

942 Figure 1. Alexander Newton Winchell mineralogist, former president of the Mineralogical  
943 Society of America (1932) and Roebling Medal recipient (1955), as taken from Emmons  
944 (1959).

945

946 Figure 2. Haüy's illustration (1801 – PL. II, figures 11 and 13) showing how the dodecahedron  
947 (left), a typical habit of silicate garnet with rhombic {110} crystal faces, is constructed by  
948 the progressive decrement of one row of *molécules intégrantes* (right).

949

950 Figure 3. Natural crystals of uvarovite (Photo Credit: Quebul Fine Minerals), grossular (Photo  
951 Credit: Peter Cristofono), andradite (Photo Credit: Dr. Didier Descouens), almandine (Photo  
952 Credit: ©JohnBetts-FineMinerals.com), spessartine (Photo Credit: Tom Loomis ©Dakota  
953 Matrix Minerals, Inc.), as well as synthetic Cr-bearing pyrope (Geiger et al., 2000) with all  
954 showing rhombic dodecahedral {110} faces (left to right and top to bottom). These garnet  
955 crystals exhibit a text-book example of isomorphs (i.e., crystals with the same form but  
956 different composition) and they form an isomorphic series (see text) or in current  
957 terminology a substitutional solid-solution series or group.

958

959 Figure 4. Illustration showing various packings, mostly of spheres, as constructed by Wollaston  
960 (1813), to account for the form of certain crystal structures. Note especially those packings  
961 (i.e., 7, 8, 9, 11,14) showing different shaded spheres.

962

963 Figure 5. Debye-Scherrer diffraction films showing reflections for the phases KBr, K(Br,Cl) and  
964 KCl and their assigned indices below (from Vegard, 1921).

965

966 Figure 6. The pyrope-grandite system as first defined by Winchell and Winchell (1927), using  
967 the data of Boeke (1914). Garnets represented by open circles and falling between the two  
968 garnet groups in this diagram were discussed by Boeke. Several of them, but not all, were  
969 considered to be poorly analyzed or represented contaminated garnets (i.e., other phases  
970 were present).

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972 Figure 7. Octahedron showing the six component silicate garnet system Py = pyrope, Al =  
973 almandine, Sp = spessartine, Gr = grossular, An = andradite, and Uv = uvarovite (modified  
974 from Geiger, 2008). Binary compositions shown by the solid lines have been synthesized in

975 the laboratory, the dotted lines represent joins where seldom-occurring natural garnets  
976 approaching the binary compositions have been found and garnets along those joins shown  
977 by dashed lines have yet to be synthesized or found in nature. Various multicomponent  
978 garnets can be found in this system or its subsystems. The molar volume for the different  
979 end-members is given in J/bar.

980

981 Figure 8. Cation-oxygen bond-length behavior as a function of the unit cell edge,  $a_o$  (Å), from X-  
982 ray single-crystal refinements on 281 silicate garnets (Merli et al., 1995). Z-O is the  
983 tetrahedrally coordinated cation-oxygen bond length, Y-O is the octahedrally coordinated  
984 cation-oxygen bond length and  $\langle X-O \rangle$  is the average dodecahedral coordinated cation-  
985 oxygen bond length of X-O(2) and X-O(4). Note the difference in the change in length  
986 behavior between PYR and GRO versus GRO and AND (PYR = pyrope, GRO = grossular,  
987 AND = andradite and SCH = schorlomite) and the gap in garnet compositions between PYR  
988 and GRO.

989

990 Figure 9. Excess volumes of mixing for six different binary aluminosilicate solid solutions,  
991 expressed using a symmetric solution model with  $W^V$ , versus a volume difference term,  $\Delta V$   
992 (as defined in the text), as determined experimentally (left - Geiger, 2000). Both linear and  
993 quadratic fits to the data (squares) are shown. Excess volumes for three different  
994 aluminosilicate garnet solid solutions, expressed using a symmetric solution model with  $W^V$ ,  
995 versus a volume difference term, here defined as  $\Delta V = V_{Large} - V_{Small}$ , as determined  
996 computationally (right - Bosenick et al., 2001). Note also the difference in units.

997

998 Figure 10. Calculated  $T$ - $X$  phase diagram for pyrope-grossular,  $(Mg_xCa_{1-x})_3Al_2Si_3O_{12}$ , solid  
999 solutions at  $P = 1$  atm (model M1-G from Dachs and Geiger, 2006).

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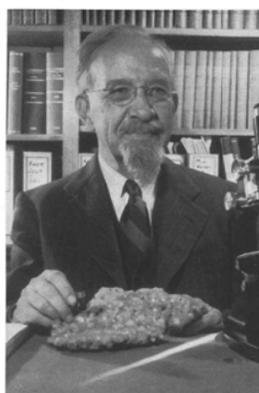
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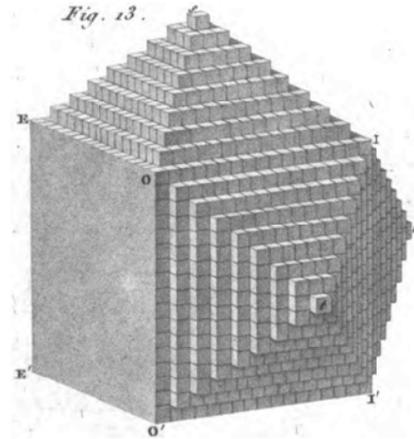
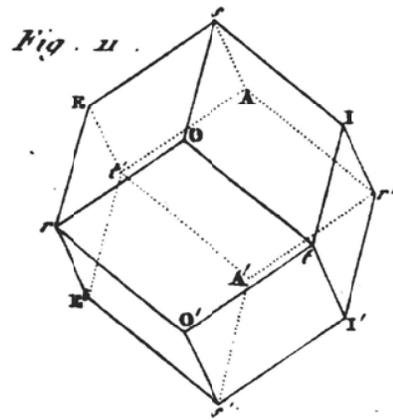


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Figure 1

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1081 Figure 3

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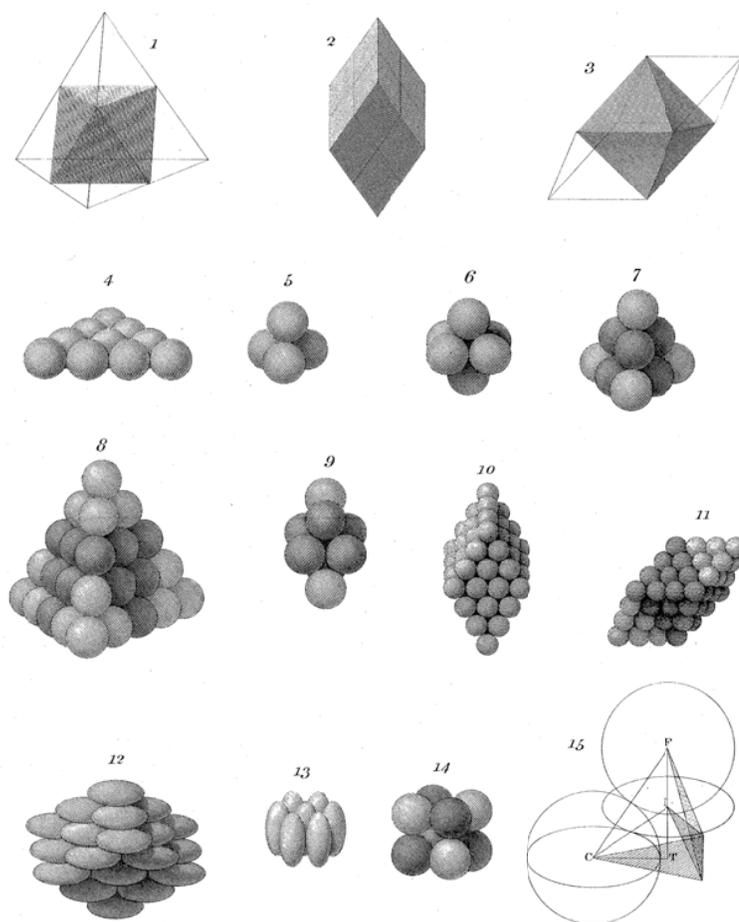
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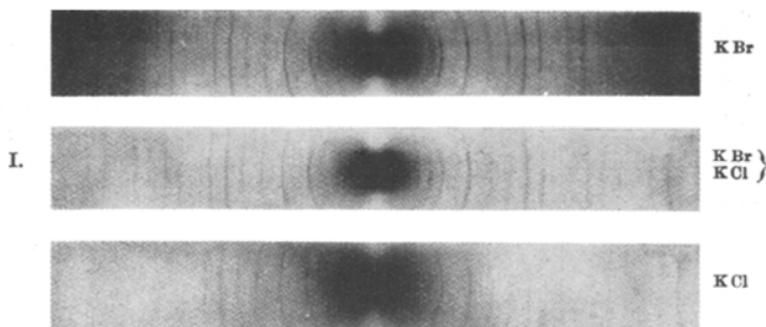
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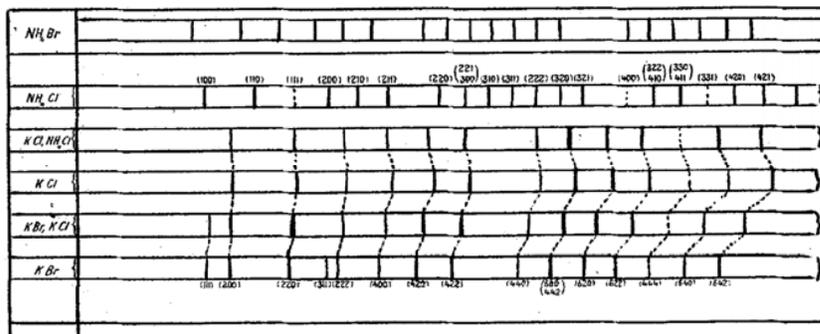
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Fig. 5

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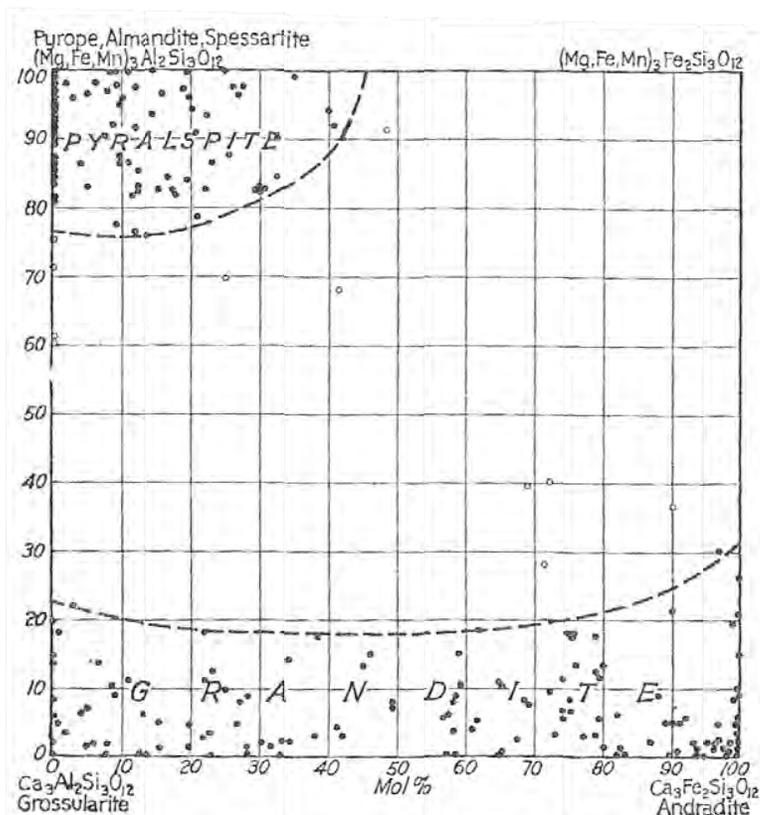


FIG. 185.—Variations in composition in garnet. Modified from Boeke: *Zeit. Kryst.*,  
LIII, 1914, p. 149.

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1139 Figure 6

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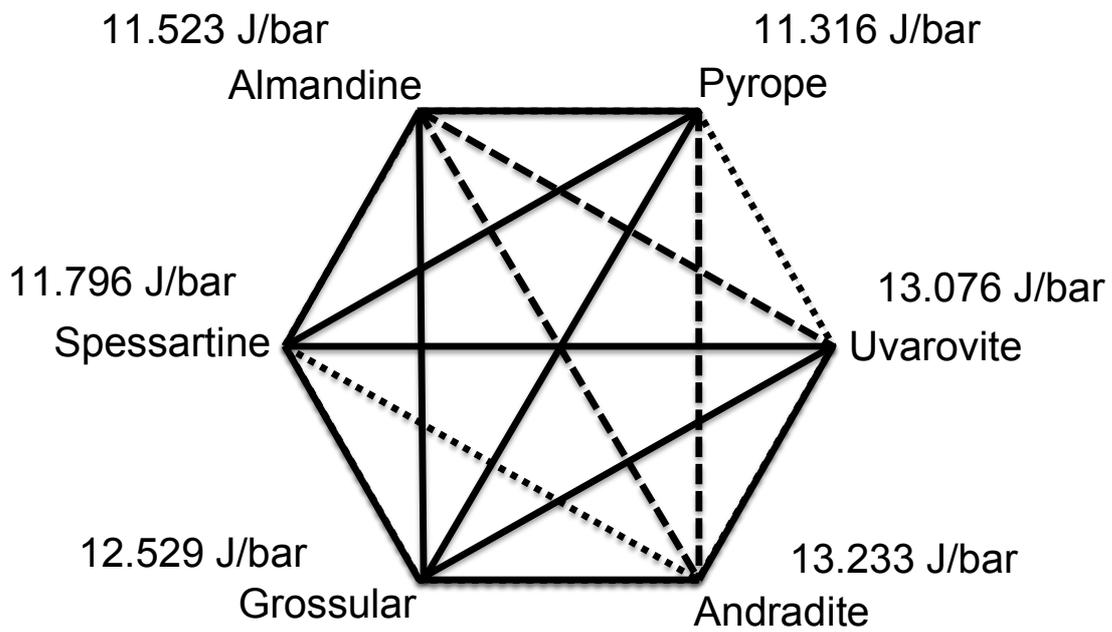
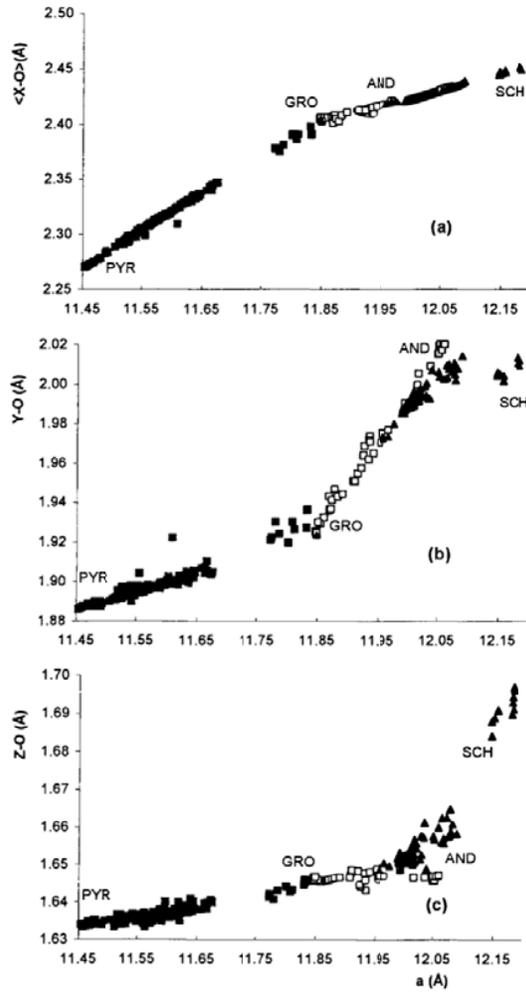


Figure 7

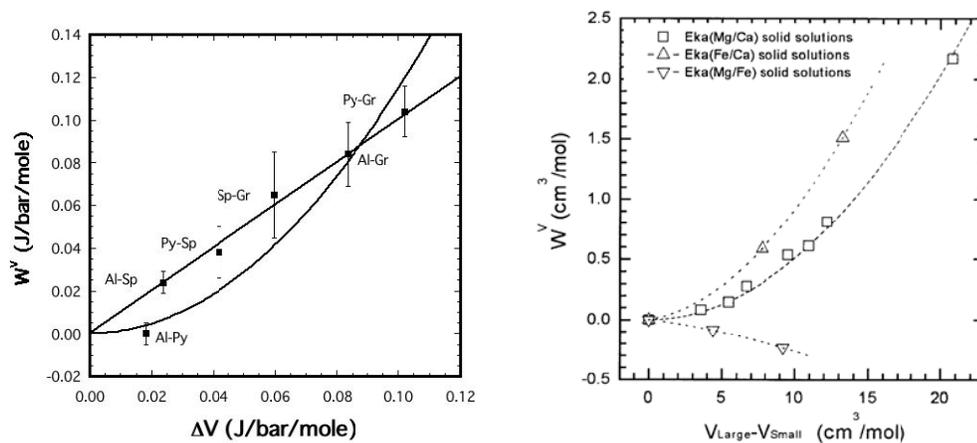
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Figure 8

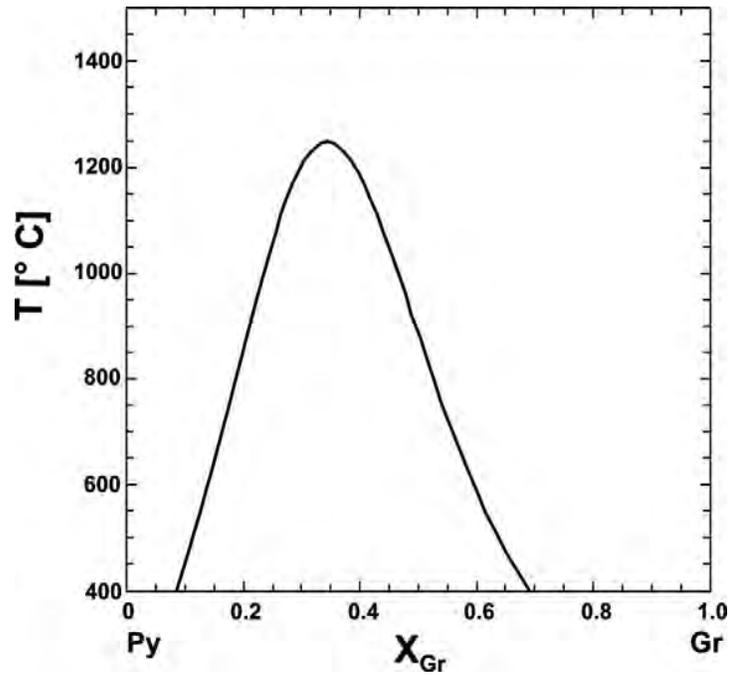
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Figure 10

Table 1. Crystal chemical formulae of selected natural garnet solid solutions calculated following Locock (2008).  
 An = andradite, Sp = spessartine, Gr = grossular, Al = almandine, Py = pyrope, Uv = Uvarovite. Remainder means unassigned.

End-Member	Formula	An-Sp #2955 <sup>1</sup>	Sp-Gr “A” <sup>2</sup>	Gr-Al 123(core) <sup>3</sup>	Gr-Al M.56.4 E (rim) <sup>4</sup>	Uv-Py S-1 <sup>5</sup>	Gr-Py-Al (table) <sup>6</sup>	Gr-Py (table) <sup>7</sup> (core)	Py-Sp (M1) <sup>8</sup>
Hutcheonite <sup>9</sup>	{Ca <sub>3</sub> }[Ti <sub>2</sub> ](SiAl <sub>2</sub> )O <sub>12</sub>	0.19%	0.54%					0.46%	0.03%
Majorite	{Mg <sub>3</sub> }[SiMg](Si <sub>3</sub> )O <sub>12</sub>					4.91%			
Morimotoite	{Ca <sub>3</sub> }[TiFe](Si <sub>3</sub> )O <sub>12</sub>					0.37%			
Morimotoite-Mg <sup>10</sup>	{Ca <sub>3</sub> }[TiMg](Si <sub>3</sub> )O <sub>12</sub>			0.78%		0.68%			
Uvarovite	{Ca <sub>3</sub> }[Cr <sub>2</sub> ](Si <sub>3</sub> )O <sub>12</sub>					45.92%	0.18%		0.26%
Goldmanite	{Ca <sub>3</sub> }[V <sub>2</sub> ](Si <sub>3</sub> )O <sub>12</sub>								0.32%
Spessartine	{Mn <sub>3</sub> }[Al <sub>2</sub> ](Si <sub>3</sub> )O <sub>12</sub>	26.94%	44.07%	3.14%	3.57%	0.75%	0.30%	2.39%	30.29%
Pyrope	{Mg <sub>3</sub> }[Al <sub>2</sub> ](Si <sub>3</sub> )O <sub>12</sub>	0.42%		0.75%	1.52%	29.88%	27.59%	35.92%	60.97%
Almandine	{Fe <sub>3</sub> }[Al <sub>2</sub> ](Si <sub>3</sub> )O <sub>12</sub>	1.17%	3.20%	43.82%	41.74%	11.57%	23.17%	0.36%	2.97%
Grossular	{Ca <sub>3</sub> }[Al <sub>2</sub> ](Si <sub>3</sub> )O <sub>12</sub>	6.56%	38.12%	47.04%	46.25%	5.58%	46.02%	58.39%	4.08%
Andradite	{Ca <sub>3</sub> }[Fe <sub>2</sub> ](Si <sub>3</sub> )O <sub>12</sub>	61.70%	3.20%	0.29%	5.58%			1.67%	

<sup>1</sup>Lee (1958); <sup>2</sup>Lee (1962); <sup>3</sup>Ackerman et al. (1972); <sup>4</sup>Ashworth and Evirgen (1984); <sup>5</sup>Sobolev et al. (1973); <sup>6</sup>O'Hara and Mercy (1966); <sup>7</sup>Sobolev et al. (2001); <sup>8</sup>Schmetzer et al. (2001); <sup>9</sup>the species hutchonite (Ma and Krot, 2014) replaces the hypothetical end-member schorlomite-Al of Locock (2008); <sup>10</sup>a hypothetical end-member composition.

Table 2. Difference in molar volume,  $\Delta V_M$ , between end-member garnets for the 15 binaries in the six component system pyrope-almandine-spessartine-uvarovite-grossular-andradite.

Garnets for those binaries in italics have neither been found in nature nor synthesized in the laboratory.

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Binary	Difference in Molar Volume (J/bar)
<i>andradite-pyrope</i>	1.918
uvarovite-pyrope	1.760
<i>andradite-almandine</i>	1.710
<i>uvarovite-almandine</i>	1.553
andradite-spessartine	1.437
uvarovite-spessartine	1.280
grossular-pyrope	1.214
grossular-almandine	1.006
grossular-spessartine	0.733
andradite-grossular	0.704
uvarovite-grossular	0.547
spessartine-pyrope	0.480
spessartine-almandine	0.273
almandine-pyrope	0.207
andradite-uvarovite	0.157

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