

DIELECTRIC BEHAVIOR OF ROCKS AND MINERALS*

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ABSTRACT

A reconnaissance study of the dielectric constants of twenty-three common mineral and seventy-one rock samples was made at thirteen frequencies in the range 50 cps to 30 megacycles. Dispersion was observed for all rock samples tested and for a few minerals at the lower end of the frequency spectrum. Moisture in a rock can increase the dielectric constant by an amount greater than is predicted by simple mixing rules. Water appears to be the principal constituent of the rock controlling the dispersion. The observed dispersion was similar in form to Maxwell-Wagner type polarization, but was quantitatively much greater. Some form of electrode or membrane polarization appears to be the most likely explanation. The dispersion is probably a result of the same mineral properties on which induced-polarization methods of geophysical prospecting are based.

INTRODUCTION

The electrical behavior of matter is controlled basically by three physical properties: electrical conductivity, magnetic permeability and dielectric constant. Values of the electrical conductivity and magnetic permeability of mineral materials and the factors controlling their variations are relatively well known. On the other hand, the dielectric behavior of matter is less studied and more poorly understood than almost any other physical property. This paper reports a reconnaissance study of the dielectric behavior of a few common earth materials, and it attempts to explain their behavior in terms of fundamental physical processes.

Dielectric "constant" is not a constant at all in the usual physical meaning of the word. It is a property which depends on such variables as temperature, pressure and frequency of alternating voltage. Fundamentally, it is the ratio of the average electrical charge displacement per unit area in a body to the electrical stress displacing it. Mathematically this is expressed

$$\bar{D} = K K_v \bar{E} \quad (1)$$

where \bar{D} is the vector electric displacement, \bar{E} is the vector electric field, K is the relative dielectric constant and K_v the permittivity of a vacuum.

It is more common to find dielectric constant defined in terms of a parallel-plate capacitor. According to this definition, the dielectric constant of a material is the ratio of the capacitance, C , of the capacitor

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with the material between the plates to the capacitance, C_0 , of a similar capacitor in vacuo

$$K = \frac{C}{C_0} = \frac{q/V}{q_0/V_0} \quad (2)$$

where the charge on the plates and the voltage between them are q and V with the material present, and q_0 and V_0 in vacuo.

The amount of charge which will exist on a pair of plates held at a given potential with respect to each other depends on the nature of the material between the plates. The dielectric constant of a material is equal to the ratio of the charge which will be present with the material between the plates to the charge which will be present in vacuo. Thus,

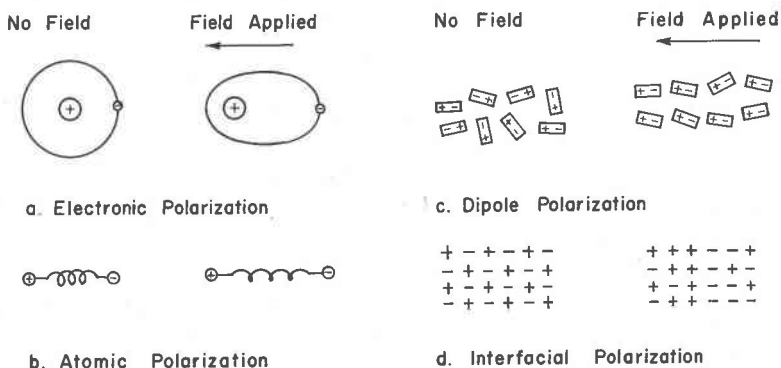


FIG. 1. Different types of polarization (after Von Hippel 1954A).

the larger the dielectric constant of the material, the larger the charge which will be present.

The charge on the capacitor plates is present because of polarization which occurs within the dielectric material. There are a number of different ways in which this polarization can occur. In all cases the polarization is a result of the relative displacement of positive and negative charges within the material. The different types of polarization generally take different lengths of time to form, which is equivalent to saying that each can be observed only below a characteristic frequency of alternating voltage. Electronic polarization (Fig. 1) results from displacements of the electron clouds surrounding an atomic nucleus. Such displacements can occur at frequencies in the visible and ultra-violet spectrum. Resonances of the electrons involved in displacements of this sort contribute to the absorption spectrum of a material.

Another cause of polarization is the motion of atoms in a crystal lat-

tice. In an electrical field the positively and negatively charged atoms are displaced from their normal positions in very small amounts. Such displacements become important at lower frequencies than for electronic polarization. Resonances occur typically in or near the infra-red spectrum.

A third type of polarization becomes important at radio frequencies. Debye (1912) postulated that the molecules of substances in which charges are not symmetrically arranged possess an electric moment which is characteristic of the molecule. When an electric field is applied, these molecules tend to align themselves with their dipole axes in the direction of the field. This is called dipole polarization. It can result also from alignment of domains or defects in a solid.

The three types of polarization considered so far are to be expected in homogeneous materials. In heterogeneous materials there is an additional type of polarization which can occur. In many materials some of the charges are relatively free to move about within the material. The degree of freedom depends on the material. In copper and aluminum some electrons are particularly free, resulting in high conductivity for these materials. In electrolytes both positive and negative ions are free to move. Even in relatively good insulators a few charges may be free to travel limited distances. If two materials adjoin in which the freedom of the charges to move differs, charges will accumulate along the interface in the presence of an electrical field. This phenomenon is called interfacial polarization. Interfacial polarization generally takes longer to form than the other types discussed. Therefore, it is rarely important above radio frequencies. Because of the effects of interfacial polarization, it is obvious that the dielectric constant of a mixture of materials may exceed that of either material alone.

As a result of these different mechanisms, dielectric constant will generally be greater at low frequencies than at high frequencies. Each of the different types of polarization adds to the dielectric constant. This is shown schematically in Fig. 2. In the case of electronic and atomic polarization, resonances are commonly sharp, so that there may be peaks and troughs in the curve. Sharp fluctuations are less common for dipole and interfacial polarizations. Ideally, the polarization takes effect asymptotically centered on a frequency known as the relaxation frequency. Figure 2 is drawn diagrammatically to bring out the separate effects. Actually the several steps of dielectric-constant increase may merge. For a heterogeneous material like a rock, it would be expected that there might be as many forms of interfacial polarization as there are varieties of interface between the various grains and contained fluids.

EXPERIMENTAL PROCEDURES

Dielectric constant was determined by measuring the capacitance of a parallel-plate capacitor with a round disc of the material being tested between the plates. To obtain these discs, plugs were cut from rock and mineral specimens using a diamond core drill. A range of sizes from $\frac{7}{8}$ " to 2" diameter was obtained, depending on the size of the original specimen. Discs from 50 to 150 mils thick were sliced from the cylindrical

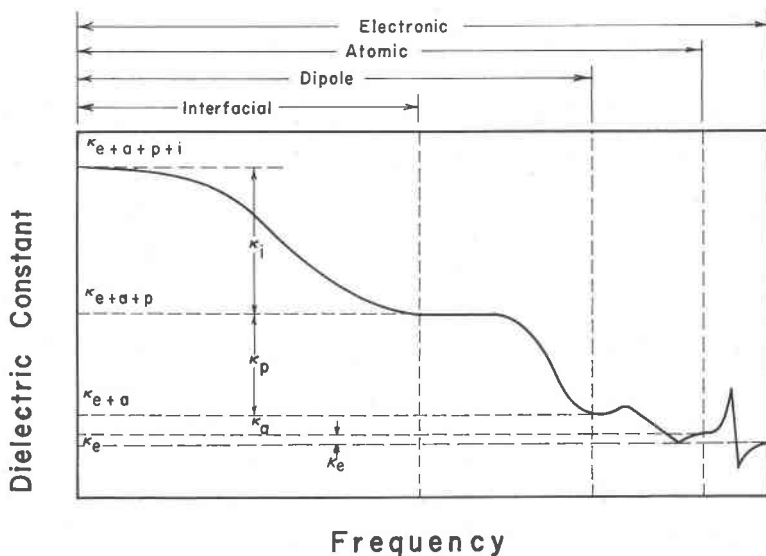


FIG. 2. Idealized graph showing the variation of dielectric constant with frequency as a result of different types of polarization.

cores using a diamond saw. The discs were ground and polished to obtain as nearly as possible smooth, parallel faces. Because of porosity, the rock surfaces generally retained some roughness.

Since failure to make intimate contact between the sample and the plates of the capacitor may result in a series air capacitance, metallic electrodes were affixed directly to the sample. One type consisted of 50-50 per cent tin-lead foil coated with petrolatum and pressed onto the sample face with a rubber roller. Under vacuum the foil sometimes loosened and separated from the sample, so that for most measurements it was replaced by a coating of conductive silver paint brushed onto the sample surface.

It is obvious from the discussion presented above that it is desirable to measure dielectric constant over as wide a range of frequencies as pos-

sible. Comparison of the measurements reported below with those of other investigators was of doubtful meaning in many cases because the frequency at which the other measurements were made was not reported. In the research described below capacitance was measured at a sequence of thirteen frequencies between 50 cycles per second and 30 megacycles. At least two measurements were made per decade. It was felt that this would permit the observation of any dispersion phenomena inasmuch as such phenomena are usually observable over several decades. Below 300 kilocycles a General-Radio Model 716-C capacitance bridge was used, with a Hewlett-Packard Model 200-C audio oscillator for a signal source and a General-Radio Model 736-A wave analyzer for a null detector. The bridge signal was monitored with a cathode ray oscilloscope. Above 400 kc. a General-Radio Model 821 Twin-T bridge with a General Radio Model 1330-A oscillator and a Collins Model 51J-3 communications receiver for a null-detector was used. The bridges were balanced both for capacitance and resistive loss. As the bridges are intended for use with low-loss capacitors only, their accuracy becomes poor for capacitors having appreciable conductance. Therefore, the dielectric constants of the more highly conductive minerals such as most sulfides and fully water-saturated sedimentary rocks can not be determined by this apparatus.

An important aspect of dielectric measurements is the means of holding the sample during the measurement. One of the main requirements placed upon a sample holder is that the configuration of the electrodes be such that the distribution of the field is known. Initial measurements were carried out using a General Radio Model 1690-A sample holder. This unit, utilizing circular-plate electrodes, is a refined model of the sample holder described by Hartshorn and Ward (1936). The electrode surfaces are precision-ground and a precision micrometer screw is used to drive the movable electrode with respect to the fixed electrode.

Early measurements on rock samples which had been oven-dried prior to measurement indicated that the samples were possibly adsorbing minute amounts of moisture from the atmosphere, which led to considerable variation in their dielectric constants during the course of measurement. To remedy this, an air-tight container was placed around the sample holder and a desiccant (P_2O_5) was put inside. Although this appeared to reduce the variability, some drift of dielectric constant was still noted. This led to the construction of an evacuable sample holder consisting of a brass plate contacting the bottom of the sample and a conical point electrode on a micrometer adjustment contacting the conductive-paint film on the top of the sample (Fig. 3).

In the case of dry samples, after cutting and applying electrodes to the

sample faces, the sample was placed in a 100° C. drying oven for a minimum of 24 hours. The sample was then removed from the drying oven and immediately placed in the sample holder. Evacuation was initiated and maintained at several millimeters of mercury for at least an hour prior to measurement. Evacuation was continued during the course of measurement. This same holder without evacuation was frequently used when measurements were made under room or natural conditions.

Where samples containing moisture were measured, partial satura-

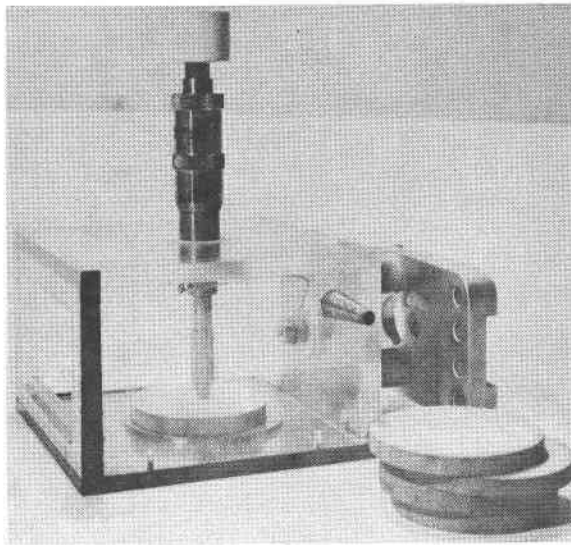


FIG. 3. Evacuatable cell for dielectric-constant measurements.

tions were obtained using evaporation techniques described by Holmes (1954, 1958). Since several per cent weight loss normally occurred during the time of measurement, the water saturation was taken as the average of the saturations determined by weighing immediately prior to and after measurement.

For measurements at elevated temperatures, the sample holder consisted of an oven (Temco Model GRP) with two stiff bare copper wires with silver tips. The sample was placed between these wires, one of which was pivoted. Temperature was measured through use of a thermocouple placed in close proximity to the sample.

Except where otherwise stated, all measurements were made at room temperature which ranged between 22° C. and 34° C.

The precision capacitors in both capacitance bridges used in this re-

search are claimed by their manufacturer to provide direct readings correct to $\pm 0.2 \mu\mu f$. When corrections supplied with the bridges are used, in conjunction with measuring differences in capacitance rather than a single direct reading, an accuracy of $\pm 0.1 \mu\mu f$ is claimed. Since the lowest value of a sample capacitance read was $5 \mu\mu f$, this represents maximum inaccuracy of 2 per cent under most circumstances. The instrument can be read to a precision of about $0.05 \mu\mu f$.

In measuring values of capacitance which fell beyond the limits of the standard bridge-balancing capacitors ($1000 \mu\mu f$), as was the case for the rocks containing bulk moisture, external variable capacitors (CDB-5, CDC-5) were required. The tolerance of these as specified by the manufacturer was ± 5 per cent.

It was possible to repeat null positions to within one half of a scale division, each division being $0.1 \mu\mu f$. A check on the over-all instrumentation was made by measuring a sample (of Kralastic EBMU) which had been previously measured at the General Radio Company and checked against their calibration standards. Their value for its dielectric constant was 4.46 as compared to a value of 4.48 obtained here.

To check the reproducibility of the measurements made on individual samples, the general practice of repeating the 50 cps measurement after each run was established. For the minerals and dry rocks, a maximum dielectric-constant variation of eight per cent was found between the two readings. In a majority of cases this variation was less than two per cent. Most or all of this variation could be due to changes occurring within the sample during measurement, such as loss or adsorption of small amounts of moisture.

For the samples containing free moisture, it was not possible to obtain such data on reproducibility, since several per cent weight loss often occurred during the course of the measurements.

To determine the combined reproducibility of both the measuring technique and sample treatment, measurements were made on three immediately adjacent samples of the Bartlesville sandstone. Each sample was exposed to the same treatment in its preparation for measurement and was measured under the same vacuum conditions. The resulting data, treated statistically, gave the most probably error of dielectric constant as .068 (out of 4.6 to 5.3) between measurements made at any given frequency. This "error" included that due to any compositional or structural variations between samples.

DIELECTRIC CONSTANTS OF MINERALS

The dielectric constants of twenty-three samples of nineteen common minerals were measured. The data are summarized in Table I and Figs.

TABLE I. DIELECTRIC CONSTANT OF MINERALS EXHIBITING LITTLE OR NO DISPERSION

Sample	Source	Cut	Dielectric Constant	
			100 cps.	10 mc.
Quartz	Brazil	perp. <i>c</i> -axis	4.59	4.51
Quartz	Brazil	parallel <i>c</i> -axis	4.60	4.50
Quartz	Brazil	45° <i>c</i> -axis	4.69	4.61
Biotite	Pontiac Co., Que.	para. bas. cleav.	4.86	4.78
Phlogopite	Denbols Twp., Que.	para. bas. cleav.	6.21	6.10
Lepidomelane	Faraday Twp., Ont.	para. bas. cleav.	4.06	3.93
Muscovite	Mattawan Twp., Ont.	para. bas. cleav.	6.74	6.60
Calcite	Unknown	perp. <i>c</i> -axis	7.60	7.60
Calcite	Unknown	parallel <i>c</i> -axis	7.30	7.30
Halite	Harshaw Chem. Co.	para. cleav.	5.85	5.85
Gypsum	Wayne Co., Utah	perp. <i>b</i> -axis	4.24	4.23

4, 5 and 6. The minerals studied were chosen on the basis of availability of samples of the most common types. Many minerals which might appear appropriate for this study are omitted because samples of sufficient purity, size or form were not available when the measurements were being made. The values of 4.5 to 4.7 obtained for quartz lie in the range

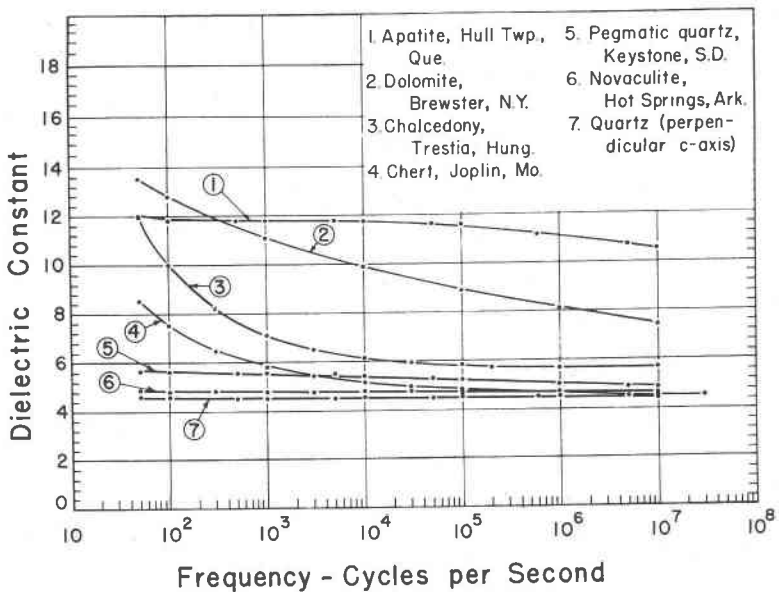


FIG. 4. Measured values of relative dielectric constant for apatite, dolomite and quartz.

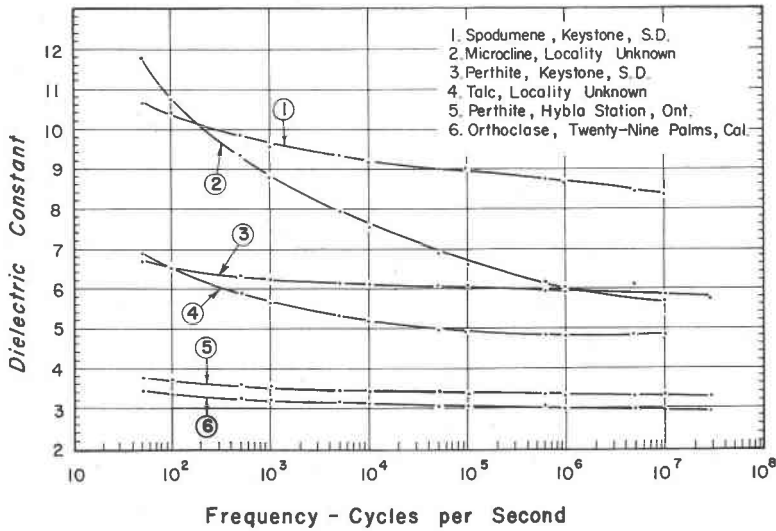


FIG. 5. Measured values of relative dielectric constant of some silicates.

of other published figures (e.g. Cady, 1946; Rao, 1947; Smithsonian Physical Tables, 1933). Novaculite, pegmatitic quartz, chert and chalcedony had dispersed, higher values of dielectric constant.

The four samples of mica tested had invariant values of dielectric constant to the accuracy of the measurements. The observed values are in the same range or lower than the values of 5.4–11.5 reported by Von Hippel (1945B) and Slichter and Telkes (1942). The dielectric constants

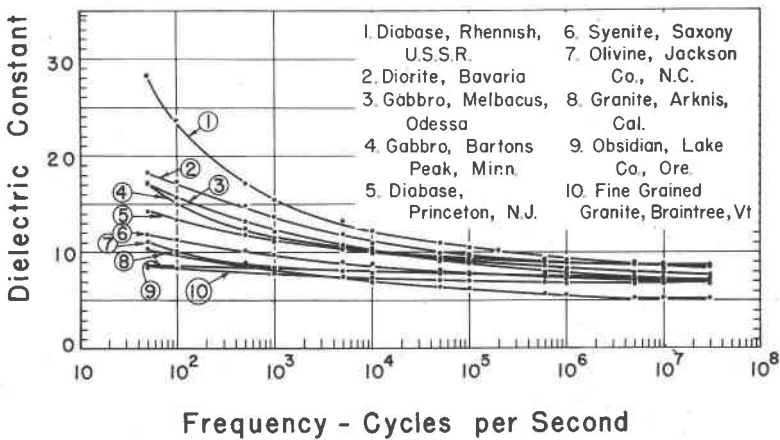


FIG. 6. Measured values of relative dielectric constant of some igneous rocks.

of the other silicates tested were all dispersed (Fig. 5). The single-crystal orthoclase showed the least and the polycrystalline bytownite the most dispersion. The values observed spanned the range of 5.45 to 7.22 reported by Takubo and associates (1953) for powdered samples of plagioclase feldspars.

The values of 7.6 and 7.3 obtained for calcite were constant, but lower than those reported by others. Rao (1949) and the Smithsonian Tables (1933) give values of 8.7 (at 1.6 mc.) and 8.49 (d.c.) for the perpendicular cut and 8.2 and 7.56 for the parallel cut. Halite and gypsum also lacked dispersion, but dolomite showed a variation. Takubo *et al.* (1953) report a value of 5.55 for the dielectric constant of halite at 1.6 mc., and 5.02 and 4.98 for two samples of gypsum at 100 mc. Slichter and Telkes (1942) and Rao (1949) quote figures of 6.3 for gypsum, 5.6 to 6.36 for rock salt (no frequency stated) and 6.8 and 7.8 perpendicular and parallel to the axis for dolomite (at 400 mc.).

Apatite had one of the most unusual dielectric constants measured (Fig. 4). Its curve was nearly flat at low frequencies and fell off at high frequencies. Slichter and Telkes (1942) quote lower but not inconsistent values of 9.5 perpendicular to the axis and 7.4 parallel to the axis at a frequency of 400 mc., which is higher than the frequency range covered here. Rao (1949) quotes figures ranging from 7.4 to 10.5 at 1.6 megacycles.

The small differences between the values reported here and those of other investigators could represent variations within the range of possible values for these minerals as a result of the presence of impurities or allowable types of structural variation; they could be the result of the different methods of measurement including the differences in the frequencies at which the measurements were made; or, less likely, they could be the effects of differences in the temperatures at which the different measurements were made.

DIELECTRIC CONSTANTS OF ROCKS

Dispersion was observed in the case of all rock samples tested. Figure 6 shows the dielectric constant of ten igneous rocks, and Fig. 7, the dielectric constant of six metamorphic rocks. The values observed at high frequencies are comparable to those reported by others (Slichter and Telkes, 1942; Rao, 1948). Marble showed the least dispersion, from 9.20 at 100 cps at 8.90 at 10 mc.

In the case of sedimentary rocks, the dielectric constant depended greatly on the moisture content. Figures for the dielectric constant of dried and natural-state Morrison sandstone samples have already been published by Keller and Licastro (1959). Similar results were obtained

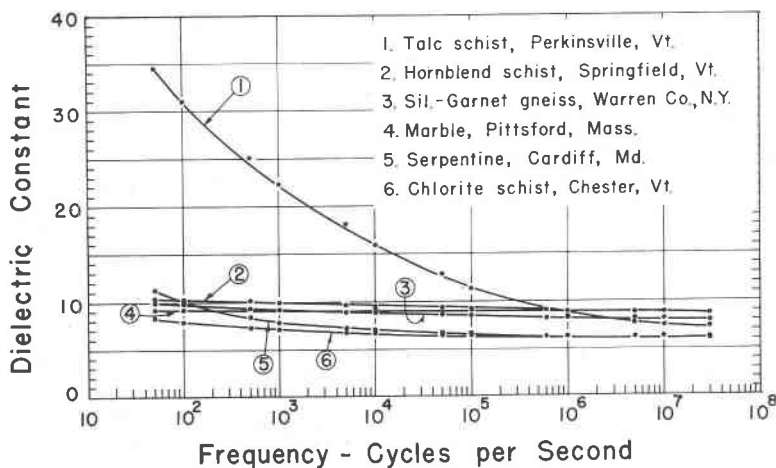


FIG. 7. Measured values of relative dielectric constant of some metamorphic rocks.

for samples of Bradford sandstone, Wilcox sandstone and Bartlesville sandstone. Even after drying under vacuum a small dependence of dielectric constant on frequency was generally observed. Values increased from 4.0–4.9 at 10 megacycles to 4.5–5.4 at 50 cps. (Compare these values with the dielectric constant of quartz of 4.5–4.7. The lower values for sandstone are expectable as a result of porosity.) Natural-state and artificially saturated samples all exhibit dispersion. Figure 8 compares the dielectric constant of a typical sample of Wilcox sandstone under natural-state (atmospheric) conditions with the same sample under vacuum and subsequent saturation with distilled water.

The dielectric constants of five typical samples of carbonate sediments are shown in Fig. 9. Values of calcite and marble are included for comparison.

FACTORS INFLUENCING DIELECTRIC CONSTANT

The factors influencing dielectric constant can be divided into three groups. First, the chemical composition of the material has an effect as shown by comparing the range of compositions among the feldspars and micas. Second, the crystal structure can be expected to influence the dielectric constant, though this is not obvious from the data reported here, as no pairs of samples of the same chemical composition and different crystal structure are included in the list. Third, the relations of the various constituents of the rocks to one another are a potential cause of variations in dielectric constant. This relationship may be partly compositional, but is almost certainly structural as well. The dielectric con-

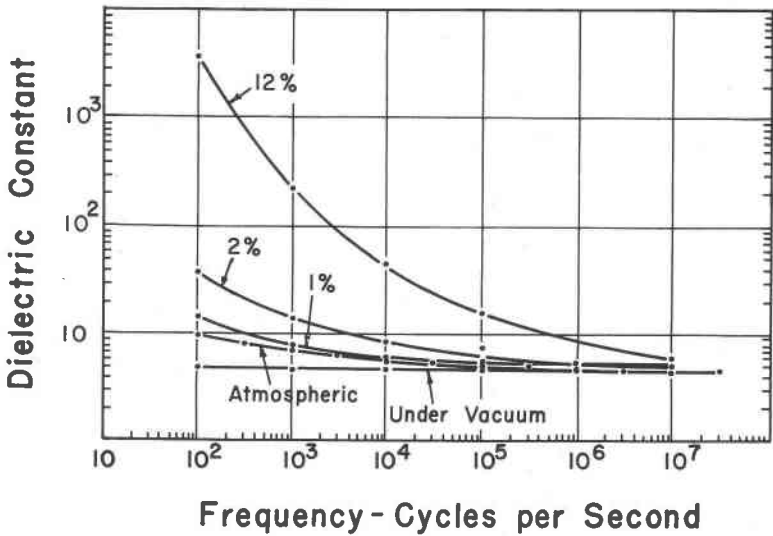


FIG. 8. Measured values of relative dielectric constant for a typical sample of Wilcox sandstone, artificially water-saturated.

stant of a rock is not generally an average of its component constituents (including the air or water filling the pores), but tends to be greater than what would be predicted by simple averaging. This increase in dielectric

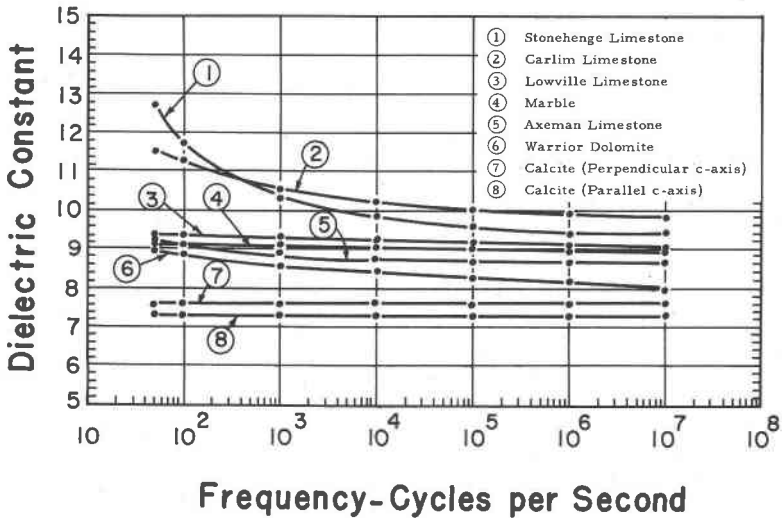


FIG. 9. Measured values of relative dielectric constant of limestones.

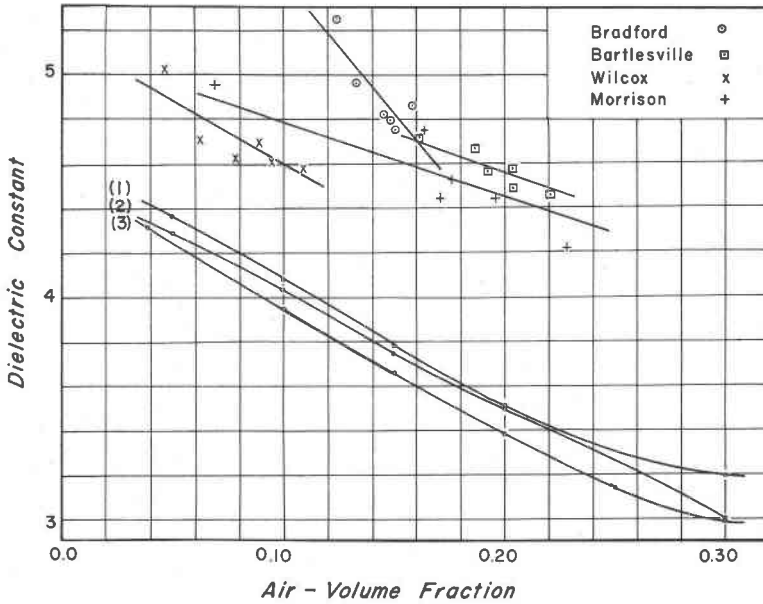


FIG. 10. Observed relative dielectric constant at 100 kc. as a function of the volume fraction of air compared to various mixing formulas.

Curve 1:
$$\frac{K - K_1}{3KV} = \frac{K_2 - K_1}{K_2 - 2K} \quad (\text{Bottcher, 1952, p. 417})$$

2:
$$\frac{K_2 - K}{K_2 - K_1} = (1 - V) \left(\frac{K}{K_1} \right)^{1/3} \quad (\text{Bruggeman, 1935})$$

3:
$$\text{Log}_{10} K = \sum V_n \text{log}_{10} K_n \quad (\text{Von Hippel, 1954A, p. 231, originally from Lichtenecker})$$

K is the dielectric constant of the mixture,

K_1 and K_2 are the dielectric constants of air and quartz, and V is the volume fraction of the quartz.

constant must be due to the interaction of the separate components present on one another.

A number of formulas have been proposed to describe the dielectric constant of simple mixtures of substances of different dielectric constant. The predictions of three of these are compared with observed values of the dielectric constants of dried sandstones at 100 kc. in Fig. 10, treating the sandstones as air-quartz mixtures. The lines through the observational data are least-squares fits. The slopes except in the case of the Bradford, a notoriously dirty sand, are roughly parallel to the predicted curves but are in all cases above them. Indeed, most of the sand-

stone samples studied had higher dielectric constants than pure quartz, even the relatively clay-free Wilcox.

The failure of mixing rules is even more striking in the case of wet samples at low frequencies. Water has a dielectric constant of about 80 under room conditions at the frequencies used here. (This is true for both pure and salt water, see e.g. Von Hippel 1954B.) Thus, moist rocks may have dielectric constants many times those of their separate components.

Examination of Figs. 4 through 9 will show that only in the case of minerals (and not all of them) is the dielectric constant independent of frequency. All rocks and some minerals show an increase in dielectric constant with decrease in frequency in the range studied. This is consistent with the observations of Takubo and Ukai (1954), who observed dispersion in the range 50–400 kc. to be related to the purity of the minerals they were testing. One and possibly the principal factor which causes this dispersion is the presence of moisture in the pores of the rock. When dried and held under vacuum, sandstone samples ceased to exhibit appreciable dispersion. Saturation with water caused an increase in dielectric constant. The amount of water needed to raise the dielectric constant is very small. A dried sample exposed to the air gradually increases in dielectric constant, presumably by the adsorption of moisture, even though the amount of water taken in is too small to measure by weighing the sample. A Wilcox sandstone sample which had been exposed to the air for a long time was put in the sealed dielectric-constant-measurement cell, and evacuated for five hours. The dielectric constant at 100 *cps* fell in about an hour from an initial value of 15.6 to an equilibrium value of 4.8. On reexposure to the air the dielectric constant gradually rose, taking around three hours to return the first third of the way toward its original value. Similar experiments with albite and diabase resulted in no drift in the dielectric-constant.

In the case of the various silica varieties tested, it is reasonable to suppose that the larger low-frequency dielectric constants of the chert and chalcedony specimens as compared to the intermediate novaculite and pegmatitic quartz values and lowest single-crystal value are due to the presence of varying amounts of water. Folk and Weaver (1952) have shown that the water content of cherts increases going from microcrystalline novaculite to "spongy" chalcedony.

Increased temperature had a similar effect. Heating a sample of Wilcox sandstone caused a rapid drop in dielectric constant from an initial value of 12.9 at room temperature to 5.5 at 100° C., then a slower, small decrease to a minimum of 5.1 at 200°, followed by a rise to 7.5 at 300°. The initial drop is explainable by loss of adsorbed water. The loss above

100° could be merely a time lag in the escape of the moisture, or could represent loss of a more tightly held water fraction. The whole heating cycle took about three hours. The rise in dielectric constant at the higher temperatures is possibly an effect of temperature on the freedom of atoms and dipoles to move in an electric field. A similar effect for quartz has been reported by Kiriyama and associates (1955). On cooling the dielectric constant fell to a minimum of 4.8 at 80° C. As room temperature was approached the rise with water adsorption again began.

The dependence on water content of the silica minerals has already been pointed out. The variation for minerals such as albite and rocks such as diabase (Fig. 6) cannot easily be explained by the presence of water. Evacuation did not significantly affect these minerals. Therefore, whatever the causes of the variation, it can occur for dry as well as moist materials.

High values of dielectric constant such as those observed here are not unique to this study. Evjen (1948), Fricke (1925, 1926), Dunlap and Makower (1945), Koops (1951), Von Hippel (1954B), Lombardini (1958) and others (see Keller and Licastro 1959) have observed comparable values for sediments in place, biological cell suspensions, dehydrated carrots and ferrites, respectively. These observers assume some form of interfacial polarization to be the most likely explanation of their observations.

The problem of mixed dielectrics was originally analyzed by Maxwell (1892) and extended by Wagner (1913). Consider a dielectric consisting of two sheets of materials 1 and 2 (Fig. 11*a*), each characterized by a relative dielectric constant, conductivity, and thickness, K_1 , σ_1 , d_1 , and K_2 , σ_2 , d_2 , respectively. Such a capacitor may be represented by the equivalent circuit shown in Fig. 11*b*. Von Hippel (1954A, 228 ff. after Maxwell 1892, Wagner 1924) has shown that a two-layer dielectric behaves as a one-layer dielectric of value:

$$K = K_\infty \left(1 + \frac{K'}{1 + \omega^2 t^2} \right) \quad (3)$$

where K_∞ is the value of dielectric constant at very high frequencies, t is the relaxation time, $\omega = 2\pi$ times the frequency and

$$K' = \frac{K_0 - K_\infty}{K_\infty} \quad (4)$$

where K_0 is the static (zero frequency) dielectric constant. From (3) and (4):

$$K = K_\infty + \frac{K_0 - K_\infty}{1 + \omega^2 t^2} \quad (5)$$

In terms of the two-layer capacitor of Fig. 11, Von Hippel has further shown that

$$K_0 = \frac{d_1 + d_2}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \left[1 + d_1 d_2 \left\{ \frac{\frac{1}{\sigma_1} \sqrt{\frac{K_1}{K_2}} - \frac{1}{\sigma_2} \sqrt{\frac{K_2}{K_1}}}{\frac{d_1}{\sigma_1} + \frac{d_2}{\sigma_2}} \right\}^2 \right] \quad (6)$$

which reduces to

$$K_0 = (d_1 + d_2) \left[\frac{K_1 d_1 \sigma_2^2 + K_2 d_2 \sigma_1^2}{(\sigma_1 d_2 + \sigma_2 d_1)^2} \right] \quad (7)$$

According to this theory, the media d_1 and d_2 may be subdivided into any number of layers without changing the dielectric response of the system, provided the volume ratios of the media are unchanged. Equation (7) can be normalized by letting $d_2/d_1 = D$ and $\sigma_1/\sigma_2 = R$, becoming:

$$K_0 = K_1(1 + D) \frac{1 + DR^2 \frac{K_2}{K_1}}{(1 + DR)^2} \quad (8)$$

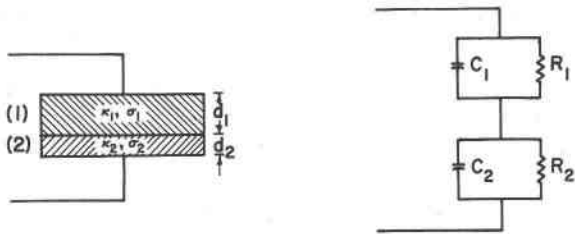
For a maximum or minimum

$$\frac{dK_0}{dR} = 2D(1 + D) \frac{RK_2 - K_1}{(DR + 1)^3} = 0 \quad (9)$$

The value $R = K_1/K_2$ corresponds to a minimum and $R = \infty$ to a maximum. When R becomes large, (8) becomes

$$K_0 \rightarrow \frac{1 + D}{D} K_2 = \frac{d_1 + d_2}{d_2} K_2 \quad (10)$$

so that K_0 is always greater than K_2 . In this case, the largest values of K_0 are reached for small values of D . Small values of D by themselves will not have a strong effect.



a. Two Layer Capacitor

b. Equivalent Circuit

FIG. 11. Maxwell-Wagner model of two-component dielectric (after Von Hippel 1954A).

What this means is that material 1 must be the more conductive material (preferably strongly so) and the effect is stronger the more plentiful material 1 is. The increase in dielectric can be visualized as a result of an effective decrease in separation of the capacitor plates resulting from the presence of conducting material between them. In Fig. 11a it is as though the charge was shifted from the upper plate of the capacitor to the interface between the layers and the conductive σ_1 -layer was shorted out.

By using the above treatment and considering the material to consist of conductive grains separated by insulating films, Koops (1951), Fairweather and Frost (1953) and Miles *et al.* (1957) were able to explain both the high values of dielectric constant and the strong dispersion of ferrites at low frequencies.

Unfortunately, this theory will not predict dielectric constants of the type observed here because conducting material is not present in sufficient quantity. If we assume the rock matrix is characterized by parameters $K_2=4.6$, $\sigma_2=10^{-6}$ mho-meters, and $d_2=0.99$, and the saturating fluid by $K_1=80$, $\sigma_1=1$ mho-meter, $d_1=0.01$ ($R=10^9$, $D=99$), K_0 will approximately equal $1.01 K_2$.

The value of the Maxwell-Wagner theory is that it shows what sort of process is needed to give a high dielectric constant. We must have some mechanism which will permit a high degree of polarization along interfaces near the electrodes, and at the same time will allow free movement of charges between these polarization surfaces.

The fact that water plays such an important role in the process suggests that electrolytic conduction is involved. The fact that only small quantities of water are involved implies that the effect must somehow be enhanced over what would occur for an ordinary conductor of the type postulated in the Maxwell-Wagner theory.

Keller and Licastro (1959) have discussed the possible role of clays in acting as semi-permeable membranes in producing polarization. If positive ions were freer to move than negative ions, then a strong charge separation could result. This theory requires the presence of clays in the rock. Since dispersion is as great for the relatively pure Wilcox sand as for the others, it appears to be inadequate to explain more than a part of the observations.

Electrode polarization could have a similar effect. Electrode polarization consists of a charge accumulation along the boundaries of metallic and ionic conductors. These interfaces could be either distributed throughout the sample or containing it. In order to measure the dielectric constant, the sample was placed between metal electrodes and a field applied. As current flowed in and out of these electrodes, charges could ac-

accumulate in their vicinity producing an effect similar to the accumulation of charges on an interface within the material being studied. Such electrode polarization, if it existed, would be due to the measuring apparatus, not to the material alone.

Keller and Licastro have rejected electrode polarization as an explanation of dispersion such as was observed here because the frequency dependence of the dispersion is not what would be predicted by postulated theories of electrode polarization. This is a negative argument. Quantitatively, electrode polarization can enhance apparent capacity sufficiently to explain the observed values of dielectric constant, as Keller and Licastro pointed out. Therefore, it can not be ruled out as at least a part of the explanation. On the other hand, the fact that strong dispersion exists for materials such as chalcedony, where such water as may be present is closely confined and is not free to migrate to the electrode surfaces, argues against electrode polarization being the only effect involved. It seems more likely that the charge accumulation is a surface effect within the rock itself, probably along water covered surfaces. The presence of dispersion in such materials as bytownite and albite suggests that significant amounts of water are not needed for the effect to occur. It may be that defects within the crystal lattice are present in sufficient quantity to allow widespread minor polarization of this sort. These defects could be water inclusions, or they could be smaller irregularities.

CONCLUSIONS

From the above discussion it can be concluded that below one megacycle, the presence of moisture in a rock plays a dominating role in determining its dielectric constant. A logical explanation of this appears to be that some form of interfacial polarization is formed, possibly along crystal or rock-fragment boundaries. Dispersion can be caused in open-structured porous rocks such as sandstones, or where the water is a more integral part of the mineral structure as in the case of chert, chalcedony and pegmatitic quartz. Water is probably not necessary for dispersion in this range, as the phenomenon was observed for minerals such as bytownite as well as for various compact igneous rocks.

It is probable that the phenomenon observed here is closely related to the induced polarization on which one variety of electrical prospecting is based (see *e.g.* Marshall and Madden, 1959). Induced polarization is known to be especially prominent in the case of sulfides and of clay bodies of certain types. The work reported here indicates that it is a very general phenomenon for natural rock formations. Strong dispersion was observed even for the "pure" Wilcox sandstone.

At low frequencies dielectric constant increases with the water con-

tent of the rock. The amount of this increase is greater than would be predicted by simple mixing formulas which have been proposed in the past. This is bound to be the case inasmuch as the dielectric constant of many rocks at low frequencies exceeds the dielectric constants of any of the components of the rocks.

A great deal more data on natural rocks needs to be gathered, especially at frequencies lower than 50 cps and with 100 per cent water saturation. Without such data, perhaps even with it, laboratory determinations of the dielectric constants of individual minerals, themselves much needed, are of little help in predicting the dielectric constants of rocks under natural conditions.

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