American Mineralogist, Volume 63, pages 893–897, 1978

Goniometry from photographs or angle-true sketches

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Abstract

A simple graphical procedure is described for preparing a gnomonic projection of a crystal from a pair of photographs or angle-true drawings related by rotation through a known angle about a known axis, and is exemplified by its application to pairs of scanning electron microscope photographs of phillipsite and calcite.

The advent of the scanning electron microscope with its great depth of focus has made it possible to obtain excellent photographs of crystals far too small for optical goniometry; it would often be of interest to ascertain their morphological development, but this is rarely possible when only a single photograph is available. However, if two photographs are available from viewpoints related to one another by rotation through a known angle about an axis the projection of which on the planes of both photographs is known, interedge angles can be derived graphically and a projection prepared with an accuracy usually of the order of 2°; if the axial elements are known (*e.g.* from X-ray data), this will commonly suffice to index the faces. Greater accuracy can be achieved if more than two viewpoints are available, and the constructions are simplified if the axis of rotation relating the photographs lies in the plane of the photographs.

The method was originally developed (Hey, 1942) to ascertain the orientation of a cut surface of an iron meteorite, and extended to drawings or photographs of crystals observed under the microscope (Hey, 1951), but is equally applicable to crystals for which optical or contact goniometry is impossible because, for example, of their situation in a cavity. It is essentially a reversal of V. Goldschmidt's admirable method of crystal drawing (1891; *cf.* Palache, 1920; Porter, 1920; Barker, 1922).

Consider Figure 1: here a drawing of willemite has been derived from a gnomonic projection, using as a guide a plan on (111)—which is essentially a drawing from a viewpoint on [111], each edge of the drawing being normal to the corresponding zone-line. The direction of any crystal edge in the final drawing is derived by joining the intersection of the zone-line representing this edge with the guide-line L (Leitlinie) to the angle-point W (Winkelpunkt); then the desired edge is at right angles to this line. The final drawing is related to the plan on (111) by a rotation around the axis A (Fig. 1) through the (stereographic) angle OW.

Clearly, if we have two drawings and know the axis and angle of rotation relating them, we can simply reverse the process and construct the gnomonic projection[1].

Of course, this example is a special case, in that one of the drawings was a plan on (111), but we can always use the same technique to prepare a gnomonic projection on the plane of one of the drawings. In Figure 2 we see this applied: the plane of the upper drawing is taken as the plane of projection, and since it is known that the two viewpoints are related by a rotation of 30° about the axis A, the guide-line L and angle-point W can be inserted. Then if a line through W normal to any edge, say [PQ], in the lower drawing cuts the guideline in P, this zone-axis is represented in the projection by a line through P normal to the edge [PQ] as seen in the upper drawing.

When all six zone-lines of this simple projection are completed, it is seen that the face P is only located by two intersecting zone-lines, but Q, R, and S are each located by three; naturally, working errors lead to error-triangles, and Q, R, and S can be located at the centroids of these. The fourth face, T, lies far out on the gnomonic projection, and the best procedure is to

[1] Strictly speaking, it is a mixed stereo-gnomonic projection, the gnomonic poles at infinity being replaced by their stereographic equivalents on the primitive circle, as advocated by Barker (1922).

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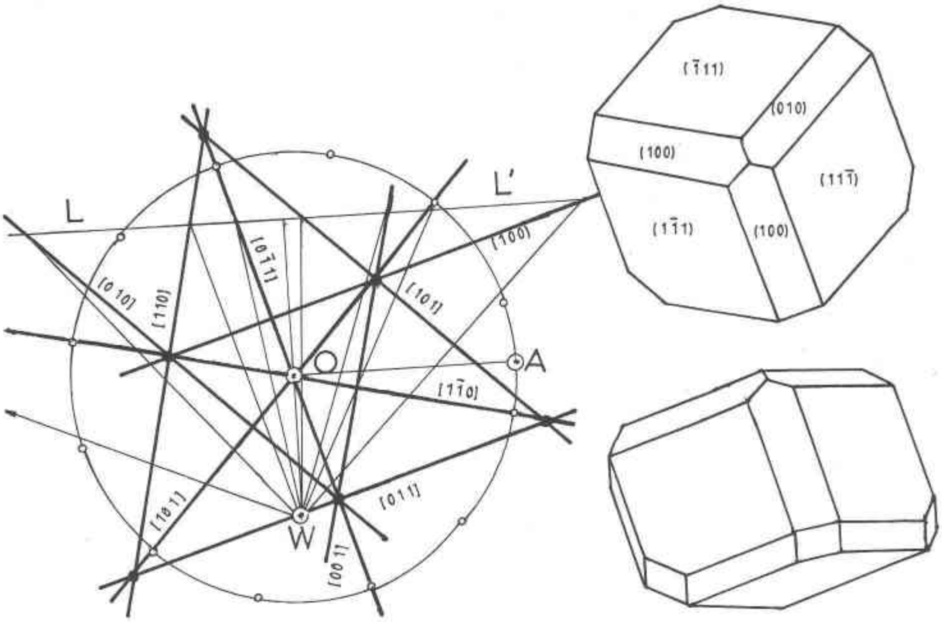

Fig. 1. Willemite: plan on (111), with a stereo-gnomonic projection and a drawing as seen from a point represented stereographically by the angle-point W. The guide-line LL′ is the trace of the plane of which W is the (stereographic) pole; AO is the axis of rotation and OW the angle of rotation (62°) relating the two views. Each edge or zone-line in the plan is normal to the corresponding zone-line in the projection (heavy lines), and each edge in the second drawing is normal to a line (light lines) joining the angle-point W to the point where the corresponding zone axis cuts the guide-line LL′. The stereographic poles of {1$\bar{1}$0} and {2$\bar{1}\bar{1}$} are shown, unlabelled, on the primitive circle.

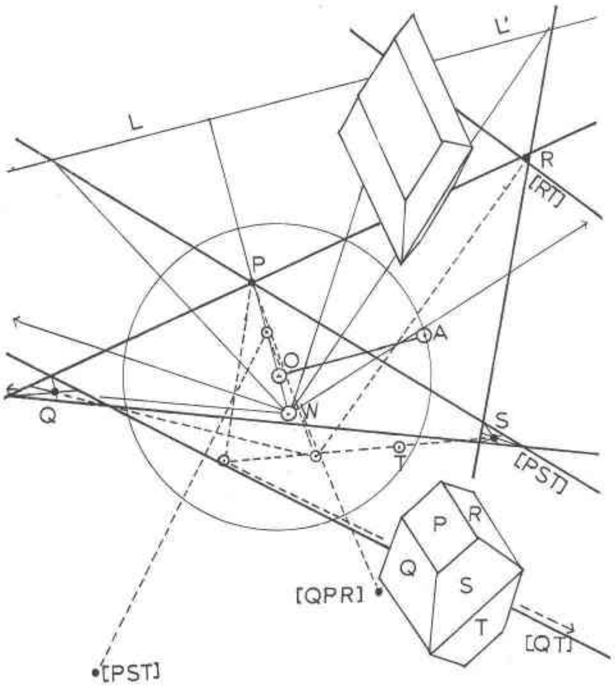

Fig. 2. Gypsum: two drawings related by a rotation of 30° about the axis OA, with a stereo-gnomonic projection on the plane of the upper drawing. In the projection the zone-lines are shown heavy and the lines for their construction light; the dashed lines are the constructions for measuring the angles PQ, PS, PR, and PT and the interzonal angle [PST]: [PQR].

measure the small angles between the zone-lines [RT], [QT], and [PST] and their normal distances from the centre of projection, O, and to calculate three estimates of the direction and central distance of T; for example, [PST] and [QT], which are at 5° to one another with central distances 24° and 33° respectively, define a pair of right-angled plane triangles in the projection with common hypotenuse OT. The three estimates of the direction of T differ by less than ½°, and those of its central distance (86°) by only 1°; T is plotted stereographically on Figure 2.

A few measurements on Figure 2 show that QR is 113° and ST 33°, and that PQ = PR and PS = PT approximately, while R$\hat{P}$S = 52°, suggesting a monoclinic crystal. In fact, the drawings are of a crystal of gypsum, and we might have guessed at monoclinic symmetry from the drawings themselves; it would then have been useful to start as in Figure 2, but after constructing the two zone-lines [PQR] and [PST] to measure the central distance of P as they define it (33°), to abandon the projection on the plane of the upper drawing, and construct instead a projection on or approximately on the presumed clinopinacoid P.

This approach is seen in Figure 4; here a twin crystal of phillipsite (BM 1932, 161, from Merri Creek, Melbourne, Australia), shown in the scanning electron microscope photographs of Figure 3, is first

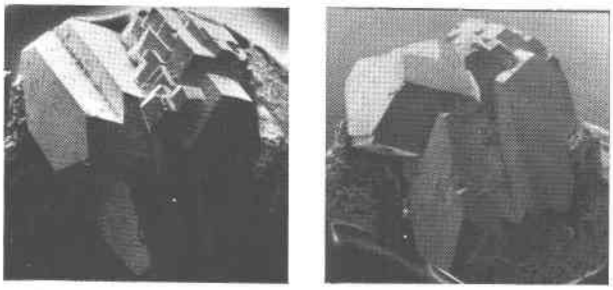

Fig. 3. Two scanning electron microscope photographs of a small group of phillipsite crystals. The photographs are related by a rotation of 51° about an axis parallel to the bottom edge of each photograph.

projected on the plane of the upper photograph of the pair, which are related by a rotation of 51° about an axis parallel to the bottom edge of the photographs, and then transformed to bring the zone-axis [PQ] vertical. Since the desired plane of projection, normal to [PQ], does not contain the axis of rotation relating the photographs, the construction for this transformation is a little more complex. The angle W_0W_1 is, of course, known, and both W_0[PQ] and $W_1\hat{W}_0$[PQ] can readily be measured from the projection (Fig. 4a); we require the angles W_1[PQ] and W_2[PQ]W_1, which can either be measured on the projection (remembering that W_0 and W_1 are stereographic poles but [PQ] is gnomonic), or calculated from the spherical triangle W_0W_1[PQ]. Then in the projection on a plane normal to [PQ] (Fig. 4b) the guide-lines L_0 and L_1 and angle-points W_0 and W_1 are readily plotted; and since the guide-lines are, in fact, the traces (linear projections) of the planes of the two photographs, their intersection is the gnomonic pole of the axis of rotation A. Therefore the two photographs must be lined up with their bottom edges (the axis of rotation) parallel to W_0A and W_1A respectively; lines normal to an edge as seen in each photograph can then be drawn through the corresponding angle-point to cut the corresponding guide-line, and the desired zone-line joins the two points so defined.

When the transformation is completed, the pseudotetragonal symmetry of the twin is clearly seen, though the face Q is badly misplaced; the zones [PQ] and [QS] meet at a very acute angle in Figure 4a, and so fail to fix Q accurately (in Fig. 4b the points on the two guide-lines are too near together, with the same result). The small face V could only be

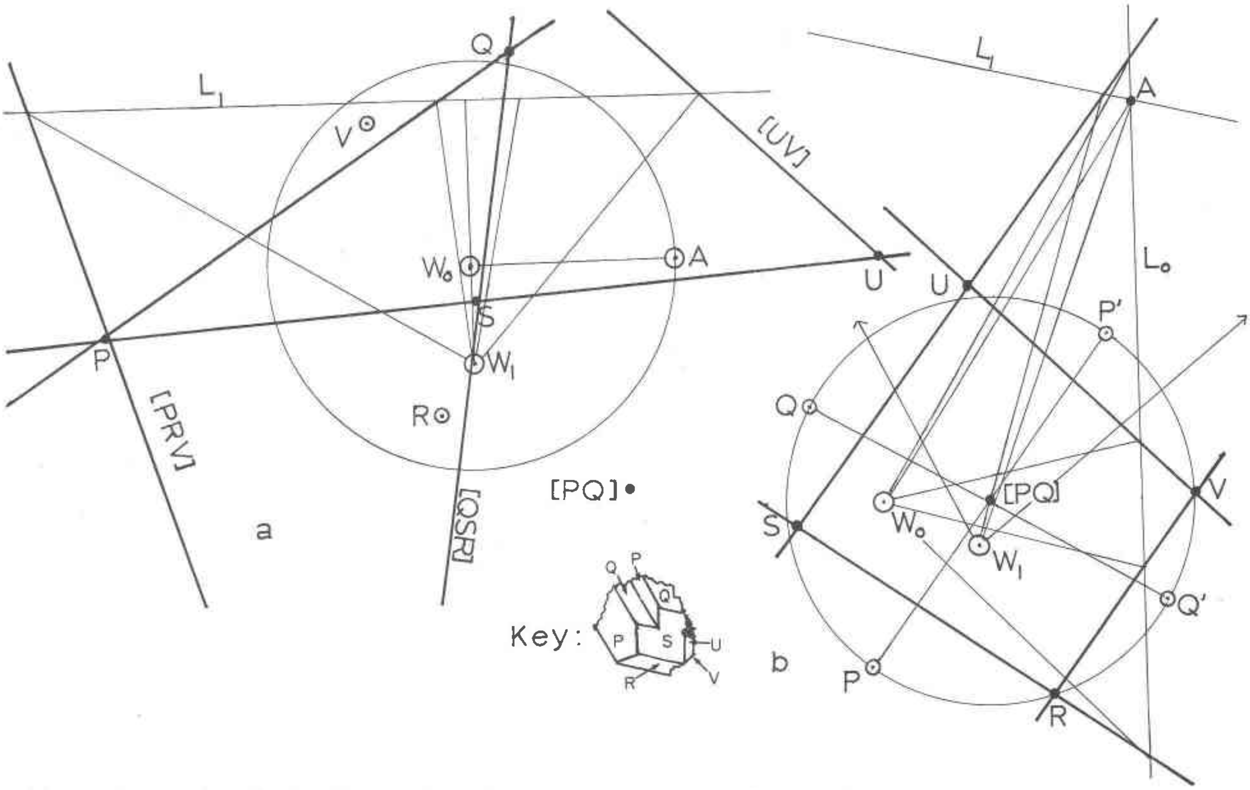

Fig. 4. a: Gnomonic projection of a crystal from the group shown in Fig. 3 on the plane of the upper photograph of the pair. b: After measuring the position of the zone-axis [PQ] on Fig. 4a, a new projection is prepared on a plane normal to this axis. The pseudotetragonal symmetry of phillipsite is clearly seen.

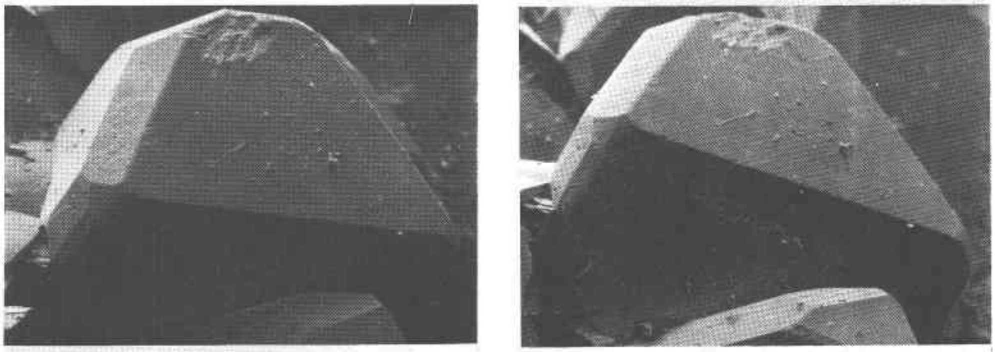

Fig. 5. Two scanning electron microscope photographs (×100) of a crystal of calcite having an unusual habit. The photographs are related by a rotation of 23° around an axis parallel to the bottom edge of the photographs.

measured roughly (that is, the edges [VU]; it is clearly in zone with P and R), but it is obviously the fourth member of the pseudotetragonal pyramid. The angle RS, measured on the projection, is 63°, reasonably close to the 60°42′ quoted by Brooke and Miller (1852).

In Figure 5 a tiny crystal of calcite from BM 1917, 750 is seen in two scanning electron microscope photographs (×100) related by a rotation of 23° about an axis parallel to the lower edges of the photographs. A projection on the plane of the upper photograph proved a little tricky to interpret, owing to the very unusual habit of the crystal, but the example serves to show the capabilities of the technique. The large face P is obviously a more-or-less equilateral triangle, and the angle PQ, measured on the projection, is 60½°,

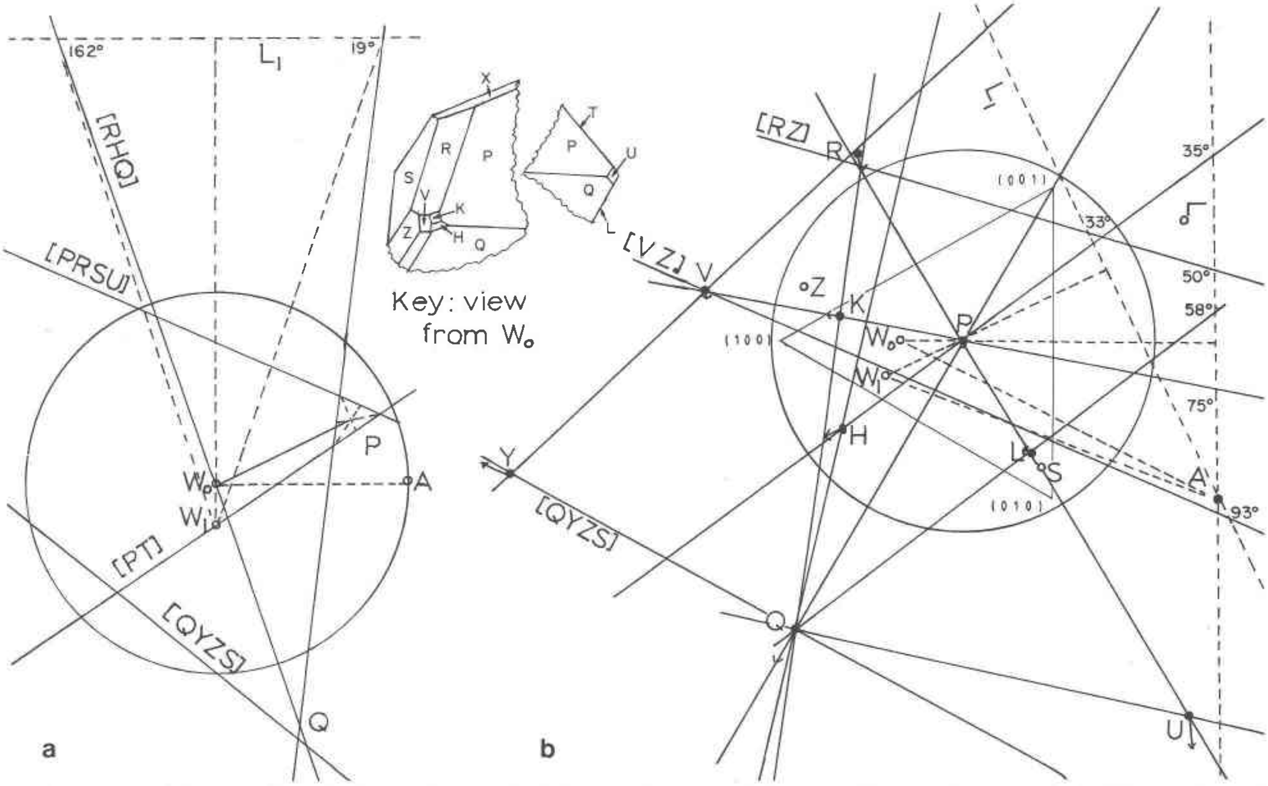

Fig. 6. a: Partial gnomonic projection of the crystal of Fig. 5 on the plane of the upper photograph of the pair. b: After preliminary measurements to establish the positions of the faces P and Q, a new projection is prepared on a plane parallel to P. Stereographic poles are shown as open circles, gnomonic as filled circles; the guide-lines and the reference lines W_0A, W_1A are shown dashed. Only those zone-lines that include a face lying far out on the projection (S or Z) are labelled. From each observed pole, an arrow is drawn to the calculated position of the face, assuming the indices given in Table 1. The small sketches on this figure and on Fig. 4 serve solely to relate the photographs to the projections.

Table 1. Observed and calculated coordinates and assigned indices for the calcite crystal of Figs. 5 and 6

Face	Obs. Phi	Obs. Rho	Calc. Phi	Calc. Rho	*
P(111)	-	0°	-	0°	o
Q($11\bar{1}$)	-60°	60½°	-60°0'	63°7'	f
L(171)	122°	35°	120°0'	33°20'	Π
V($5\bar{1}0$)	11°	54°	8°57'	53°56'	σ
R($5\bar{3}\bar{5}$)	58°	47½°	60°0'	48°26'	
Y($20\bar{1}$)	-16°	68°	-13°54'	69°2'	v
U($\bar{1}4\bar{1}$)	-122°	66½°	-120°0'	67°56'	
S($\bar{1}3\bar{1}$)	-122°	77°	-120°0'	75°47'	m
Z($6\bar{4}\bar{1}$)	18°	83°	17°0'	83°35'	
H($42\bar{1}$)	-36°	38½°	-38°13'	40°42'	
K(601)	11°	33½°	8°57'	38°7'	q

* Significant letter after Brooke and Miller

suggesting that P is (111) and Q ($11\bar{1}$); Figure 6a is an incomplete projection on the plane of the upper photograph, designed to locate P and Q as accurately as possible, so that the distance PW_0 and the angle $W_0\hat{P}W_1$ can be measured or calculated, and a projection on P prepared (Fig. 6b). Assuming that Q is ($11\bar{1}$), the primitive gnomogram of calcite can be drawn [the triangle (100):(010):(001)]. The stereographic poles (angle-points) of the two viewpoints are plotted (W_0 and W_1) and their guide-lines L_0 and L_1 (dashed lines), intersecting in the gnomonic pole of the axis of rotation, A. The two photographs are lined up parallel to the lines W_0A and W_1A and the positions where normals to the several edges cut the appropriate guide-lines marked; in practice, it proves more convenient to measure the angles between the interfacial edges and the bottom edges of the photographs (the axis of rotation), and use these angles to set off the proper points on the guide lines, measuring about the appropriate angle-point and taking the lines W_0A and W_1A as 90°. In this way the faces R, Y, and V are readily located; S and Z lie far out on the projection, but their positions can easily be calculated from the appropriate plane triangles. The faces U, L, and H each lie in one well-defined zone ([RPSLU] and [QH]); the edges [QU], [QL], [PH] could each only be measured reliably on one photograph, and were therefore drawn through the established faces P and Q, as is the zone-line [PV]. The remaining small face, K, appears to lie truly in zone [VP], and nearly but not quite in zone [RQ] or [RH]. It is now possible to index all the faces with reasonable certainty; the indices are shown in Table 1, with their observed and calculated coordinates. (The face X is visible on one photograph only; as the edge [RX] could not be measured reliably, this face could not be projected or indexed.)

The procedure is not, of course, confined to pairs of photographs. For example, it has been used for a series of photographs of a crystal taken with an ordinary camera, the specimen being mounted on a simple turntable the axis of which was at 30° to the optic axis of the camera, and could equally well be applied to pairs of camera lucida drawings. It can obviously be useful in other situations where crystals are either too small or too inaccessible for optical or contact goniometry, and could be extended to the well-developed crystals obtained in some microchemical tests on slides, the slide being mounted on a U-stage, though correction for refraction at the surface of the immersion liquid would be necessary.

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Manuscript received, March 29, 1978; accepted for publication, May 15, 1978.