

## Supplementary Appendix II

Hålenius et al. (2007) undertook an analysis of the Racah parameters  $B$  and  $C$  and the crystal-field-splitting parameter,  $\Delta = 10Dq$ , by considering the spectra of a number of  $\text{Mn}^{2+}$  phases including the spinel galaxite ( $\text{Mn}^{2+}\text{Al}_2\text{O}_4$ ). They labeled the various  $\text{Mn}^{2+}$  spin-forbidden bands, typically five in number, from  $\nu_1$  to  $\nu_5$  (see Table 4 in Hålenius et al. 2007) with increasing wavenumber and gave exact equations to calculate the energies of  $\nu_3$  and  $\nu_5$  and approximate solutions for  $\nu_1$ ,  $\nu_2$  and  $\nu_4$ . In terms of galaxite,  $B$  and  $C$  can be calculated from their eqns. (3) and (5) and  $Dq$  obtained from their eqns. (1) and (2) and in doing this they obtained  $B = 627 \text{ cm}^{-1}$ ,  $C = 3424 \text{ cm}^{-1}$  and  $Dq = 592 \text{ cm}^{-1}$ . We note that eqns. (1) and (2) are not real in a strict sense, because if  $Dq = 0$ , the wavenumber of both  $\nu_1$  and  $\nu_2$  would be infinite in value, instead of the case where both would have equal energies corresponding to the electronic term  $^4G$  (e.g., Burns 1993, Fig. 3.10). Taking the  $B$ ,  $C$  and  $Dq$  values for galaxite and inserting them into their eqns. (1) and (2), one calculates  $\nu_1 = 19167$  and  $\nu_2 = 23389 \text{ cm}^{-1}$ . If, alternatively, one calculates them from the energy matrices of Tanabe and Sugano for the  $^4T_1$  and  $^4T_2$  states of the electronic configuration  $d^5$  (i.e., Sviridov et al. 1976) and assuming a Trees parameter of  $\alpha = 0$ , one obtains  $\nu_1 = 19745$  and  $\nu_2 = 22145 \text{ cm}^{-1}$ . These values agree better with the experimental spectroscopic values for these transitions of 20300 and 22250  $\text{cm}^{-1}$ , respectively. Note also that when  $Dq = 0$ , one has  $\nu_1 = \nu_2 = 23390 \text{ cm}^{-1}$  for the electronic term  $^4G$ . Similarly, the calculated wavenumber of 26529  $\text{cm}^{-1}$  for band  $\nu_4$  (i.e., the  $^6A_1(S) \rightarrow ^4T_{2g}(D)$  transition) in galaxite, as obtained from its energy matrix, is closer to the experimental value 25970  $\text{cm}^{-1}$  compared to the value of  $\sim 25271 \text{ cm}^{-1}$  calculated from eqn. (4) of Hålenius et al. (2007). However, in spite of the various differences, if one considers the uncertainties associated with crystal-field and Racah-parameter

theory, we think eqns. (1), (2) and (4) give roughly similar wavenumbers to those calculated using energy matrices.

Assuming that the wavenumbers of the two field-independent electronic transitions  ${}^6A_1 \rightarrow {}^4A_1, {}^4E ({}^4G)$  (band  $\nu_3$  of Hålenius et al. 2007), as given by the mean wavenumber value for bands  $i$  and  $j$ , and that the wavenumber value of band  $m$  (i.e.,  $\nu_5$  with  ${}^6A_1(S) \rightarrow {}^4E(D)$ ) - see Table 2 - we obtain for our Mn gahnites  $B = 561$  and  $C = 3530 \text{ cm}^{-1}$ . We also calculated the wavenumber for the two field-dependent electronic transitions  ${}^6A_1 \rightarrow {}^4T_1(G)$  and  $\rightarrow {}^4T_2(G)$ , labeled  $\nu_1$  and  $\nu_2$ , respectively (Hålenius et al. 2007), in the gahnites studied here, using their energy matrices (Sviridov et al. 1976). Ideally, the calculated wavenumber values of  $\nu_1$ ,  $\nu_1$  and  $\nu_4$  should agree with their experimental values for the same value of  $Dq$  (c.f. Burns 1993 - Fig. 3.16). This is always the case for  $\nu_3$  and  $\nu_5$ , because they are field independent (i.e., a function of  $B$  and  $C$  but not of  $Dq$ ) and  $B$  and  $C$  are calculated from their energies. However, we obtain an equivocal result for  ${}^4T_1(G)$  and  ${}^4T_2(G)$  that best agree with the experimental values of 20730 and 21910  $\text{cm}^{-1}$  for rather different values of  $Dq$ , that is, 472 and 596  $\text{cm}^{-1}$ , respectively. Adopting a mean intermediate value of  $Dq = 534 \text{ cm}^{-1}$ , one calculates  $\nu_1 = 20213$  and  $\nu_2 = 23251 \text{ cm}^{-1}$ . They differ significantly from the experimental values of 20730 and 21910  $\text{cm}^{-1}$  (i.e., +517 and -1305  $\text{cm}^{-1}$ , respectively).

Concluding, we think that the noted disagreements between experiment and theory in our analysis are mainly due to the fact that the tetrahedrally coordinated  $\text{Mn}^{2+}$  in our gahnites is distorted from regular tetrahedral symmetry. This proposal is supported by the observed splitting and fine structure of the electronic bands  ${}^6A_1 \rightarrow {}^4T_2$  and  ${}^6A_1 \rightarrow {}^4A_1, {}^4E ({}^4G)$  shown in the spectra of Fig. 6 (see also Table 2). Therefore, the energy matrices that assume cubic symmetry (Sviridov et al. 1976) are not applicable. Moreover, we think that the Racah parameters  $B$  and  $C$

probably cannot be determined from the wavenumbers of the strongly split  ${}^6A_1 \rightarrow {}^4A_1, {}^4E ({}^4G)$ , i.e. i and j bands, and band m (Fig. 6) assigned to  ${}^6A_1(S) \rightarrow {}^4E ({}^4D)$ . It is also of note that the ratio  $C/B \approx 6.3$  significantly differs from the “typical value” of 3.5 obtained for various  $Mn^{2+}$ -bearing phases (e.g., Burns 1993 - Table 11.1). This indicates that the Racah parameters  $B$  and  $C$ , based on our analysis and the use energy equations for  $\nu_3$  and  $\nu_5$  (Hålenius et al. 2007), can not be determined quantitatively.